

LEARNED INDEXES FROM THE ONE-DIMENSIONAL TO THE MULTI-DIMENSIONAL SPACES: CHALLENGES, TECHNIQUES, AND OPPORTUNITIES

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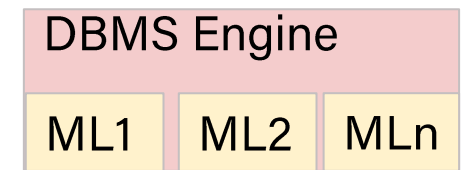
Outline

- Introduction and Historical Background
- Learned Indexes for the One-dimensional Space
- Learned Indexes for the Multi-dimensional Space
- Open Challenges for Future Research

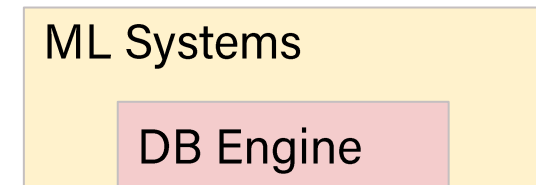
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Introduction: Database Systems + Machine Learning

- Machine Learning for Database Systems (ML for DB)
 - Replace core components of a Database System (e.g., query optimizer, Indexes, DB administration) with Machine Learning techniques
 - Achieve better performance
 - Less space requirement
- Database Systems for Machine Learning (DB for ML)
 - Extend database system techniques to support efficient ML workloads



ML for DB



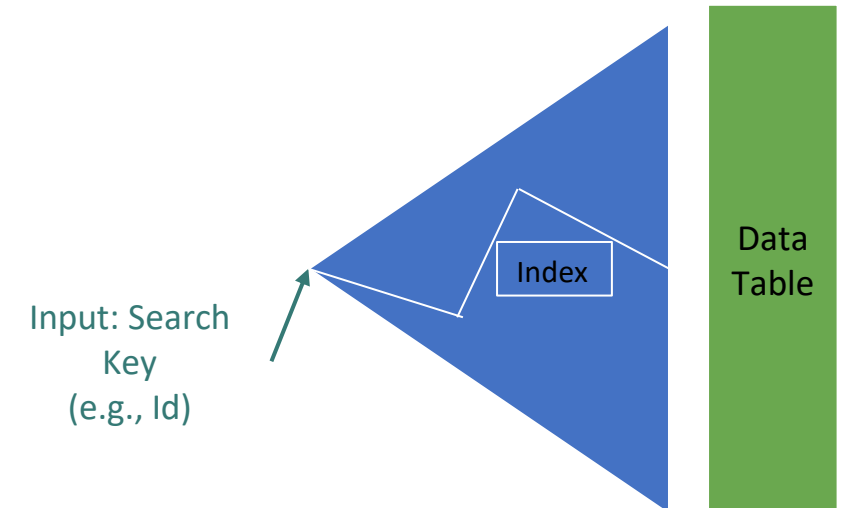
DB for ML

Introduction: Data Lookup

- Given a large collection of data, there are multiple ways for data lookup:
 - Linear Search
 - Binary Search if data is sorted
 - Build Index structures
- *What if we have a good estimation over the data distribution in advance?*
 - *Can we further speed up the process of data lookup?*

Introduction: Database Indexing

- Database Index: Provide efficient access to data
- Popular index structure: B⁺-tree
- Given search key, B⁺-tree identifies the storage location of the tuple that contains the search key
- Can view the B⁺-tree as a function: B⁺-tree(key) → position
 - Take key as input and return the order of key's tuple inside the table

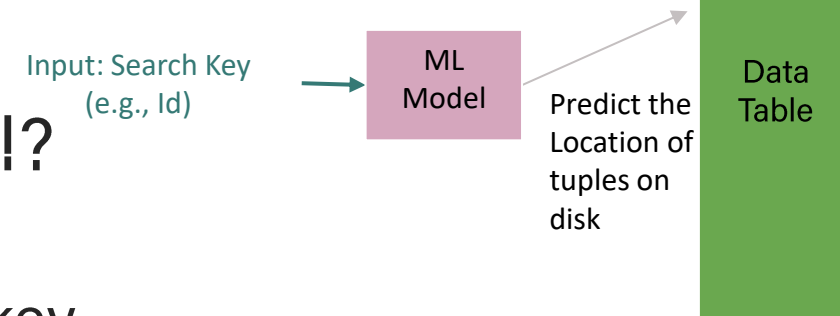


Introduction: One- vs. Multi-dimensional Index

- **One-dimensional Index**
 - Built on dataset with only a single dimension
 - Data can be sorted (i.e., totally ordered).
- **Multi-dimensional Index**
 - The dimension of the data is typically between 2 to 10.
 - No obvious total sort order for multi-dimensional data
 - In contrast, dimension of the data can be very high (e.g., 100) in the context of high-dimensional index.

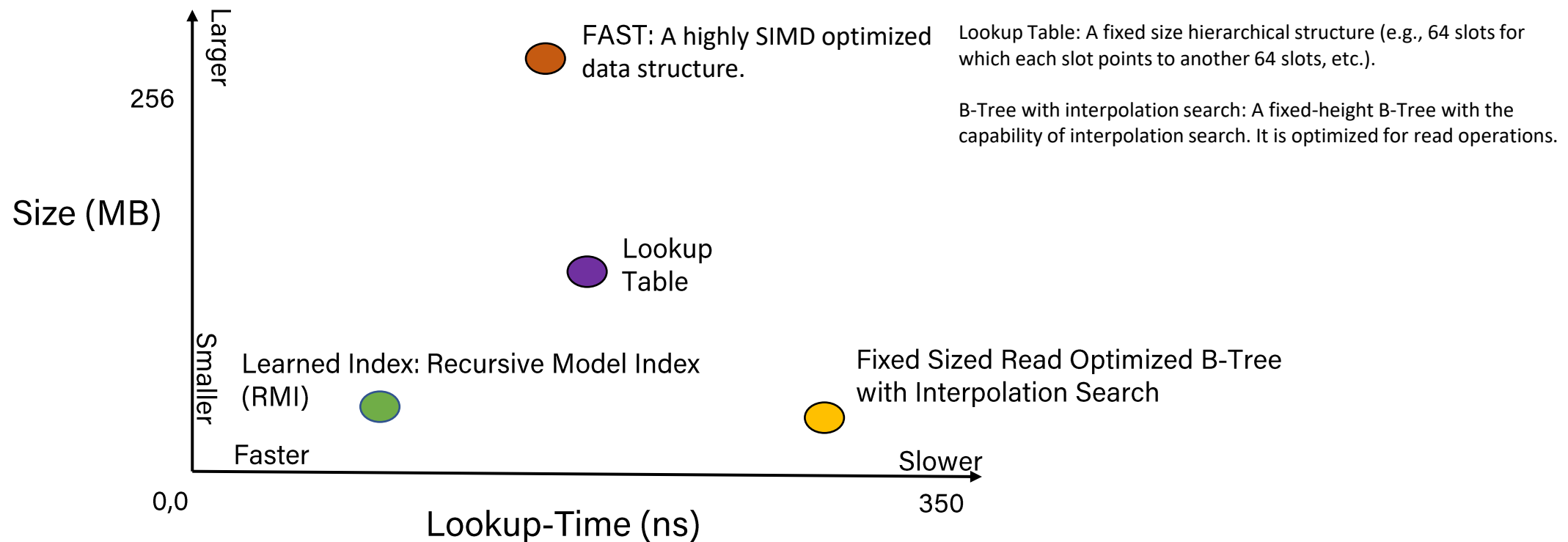
Introduction: Learned Indexes

- Can one use ML techniques to guide data indexing?
- Can we learn the function:
 - B^+ -tree(key) $\rightarrow$ Location of tuple in table?
- Can we replace the B^+ -tree with an ML model?
 - Learned Index: viewing an index as a model
 - ML-Model(key) predicts the storage location of the key
 - Include an error correction mechanism (e.g., binary search) if the model mispredicts
 - Feasible approach due to the **total order** of the one-dimensional data
 - For example, with a simple linear model, we only need two floats: slope and intercept
 - Searching executes potentially in $O(1)$ time
 - Space requirement is also potentially $O(1)$



Introduction: Initial Results

- Initial performance results (approximate) of a learned index
 - Promising → Faster lookup time and smaller storage



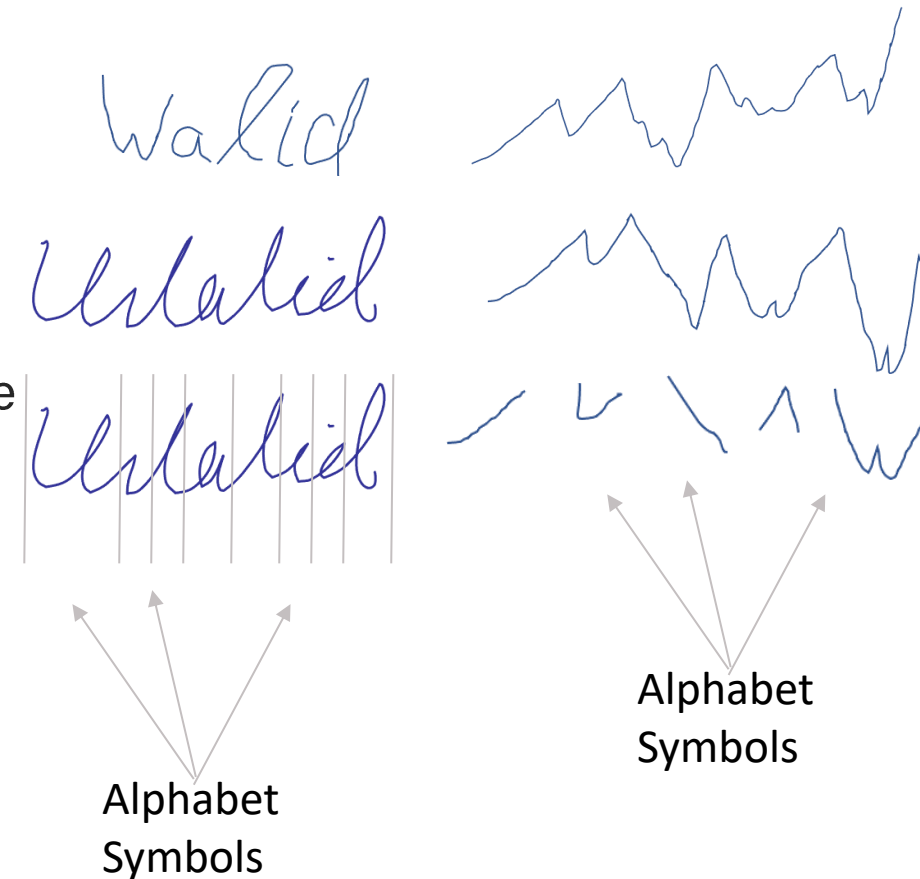
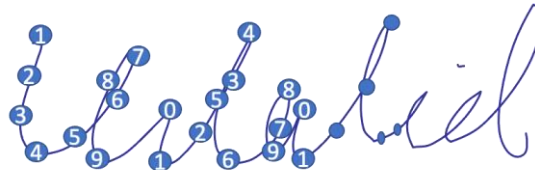
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Introduction: Indexing the Learned Models vs. Learning the Index

- Indexing the Learned Models
 - Given a collection of ML models, e.g., object recognition models (Cats, Dogs, Trains, etc.), and an input object, say o
 - Identify the class of o (cat vs. dog, etc.)
 - Instead of executing all models and identifying which has the highest matching score
 - Can we index the learned models to speed up the matching process?
- Learning the Index (i.e., Learned Indexes)
 - Given a key value, say k , and an ordered array of key values
 - Build an ML-based model that helps predict the location of k in the ordered array

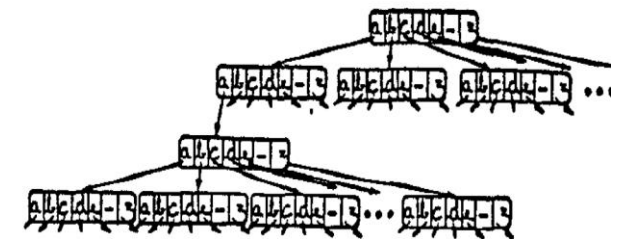
Introduction: Indexing the Learned Models

- Collection of spatiotemporal sequences, e.g., heart pulse rates, stock market trends over time, handwritten drawing on a tablet, object movement trajectory
 - Index shown in the context of handwritten text
- Divide each spatiotemporal sequence into basic alphabet symbols
- Because of the variability, there is a need for training to recognize similar, but not exactly the same, patterns.
- Model each alphabet symbol in the spatiotemporal sequence using *local spatiotemporal features* along the trajectory of the sequence
 - Time
 - Velocity
 - Direction
 - Acceleration
 - Aspect ratio, ...



Introduction: Indexing the Learned Models (Cont'd)

- Left-to-right Hidden Markov Models are suitable for representing spatio-temporal sequences
- Instead of building a Hidden Markov Model for each entire sequence, we build an HMM for each alphabet symbol in the spatiotemporal sequence
- Need to segment each spatio-temporal sequence into alphabet symbols
- Train the left-to-right Hidden Markov Model using multiple samples of the alphabet symbols
- Indexing the learned Hidden Markov Models
- Construct a trie structure over the learned alphabet symbols



Introduction: Indexing the Learned Models (Cont'd)

Hidden Markov Model

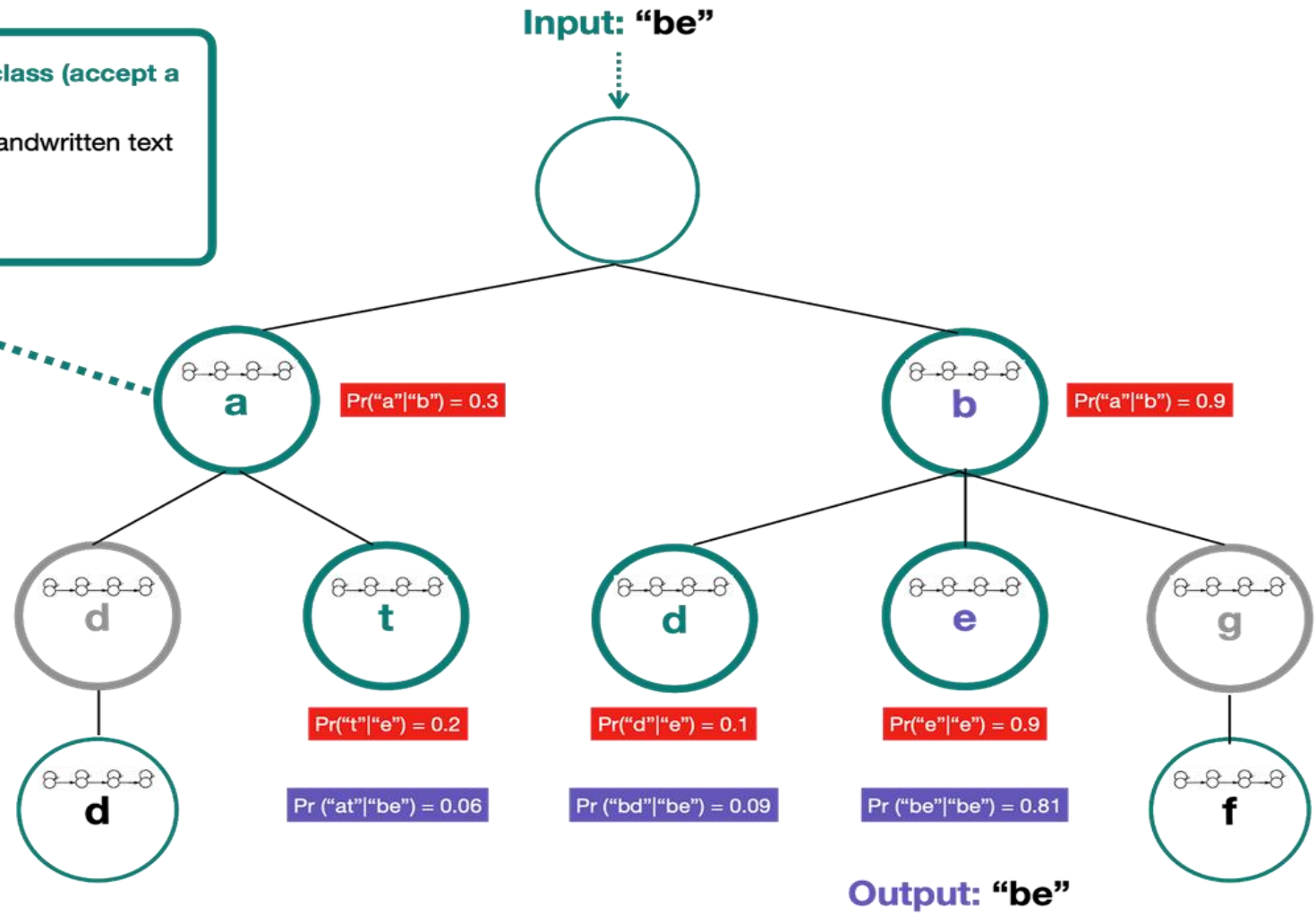
Each HMM is constructed to represent a pictogram class (accept a specific pictogram with high probability)

- **Left-to-right HMM:** Useful for modeling cursive handwritten text
- **Input:** Pictogram
- **Output:** Matching Probability

Traverse Nodes (HMM Models)

Choose Nodes with highest probability

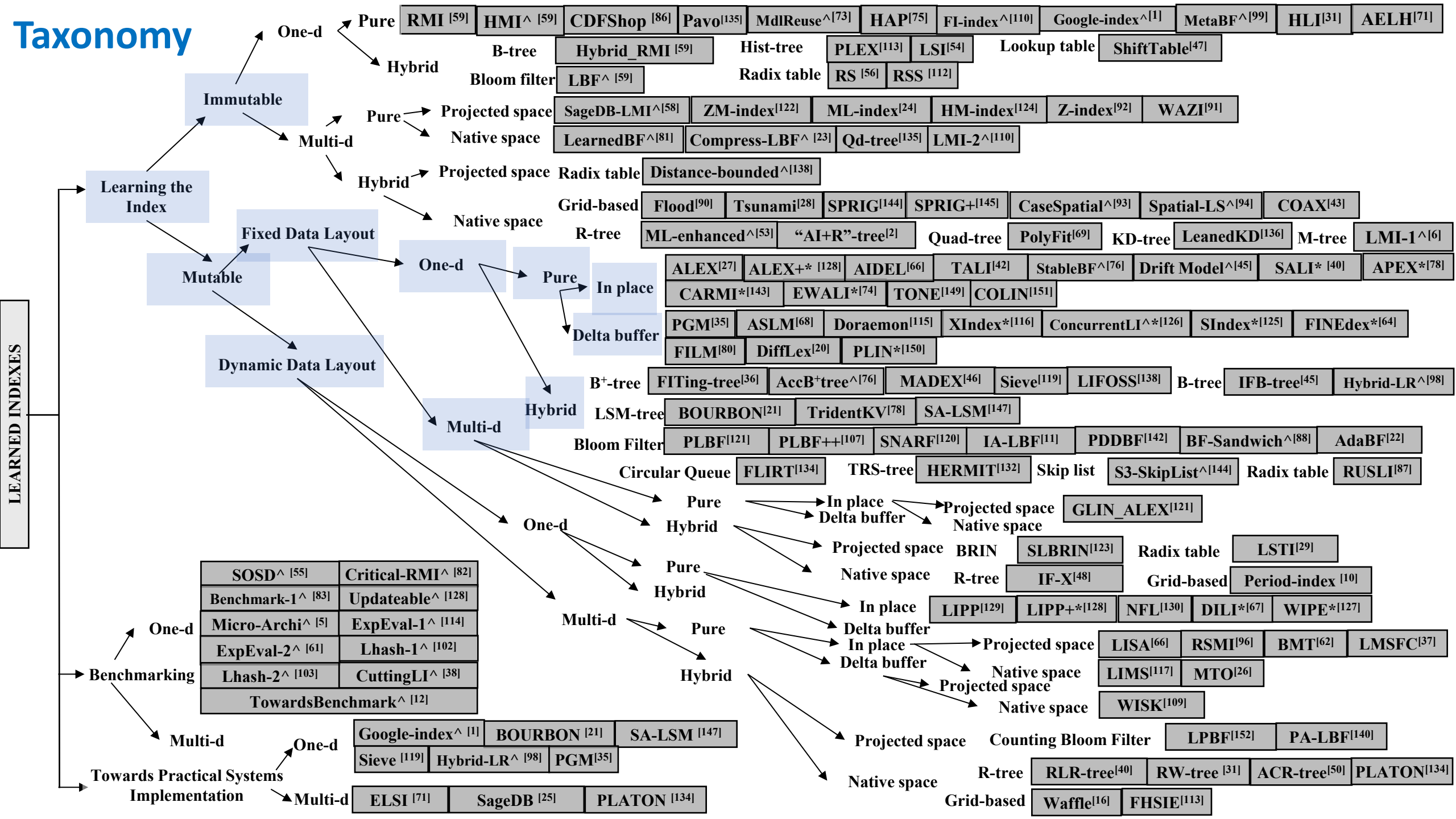
choose combination of nodes with highest probability



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Taxonomy



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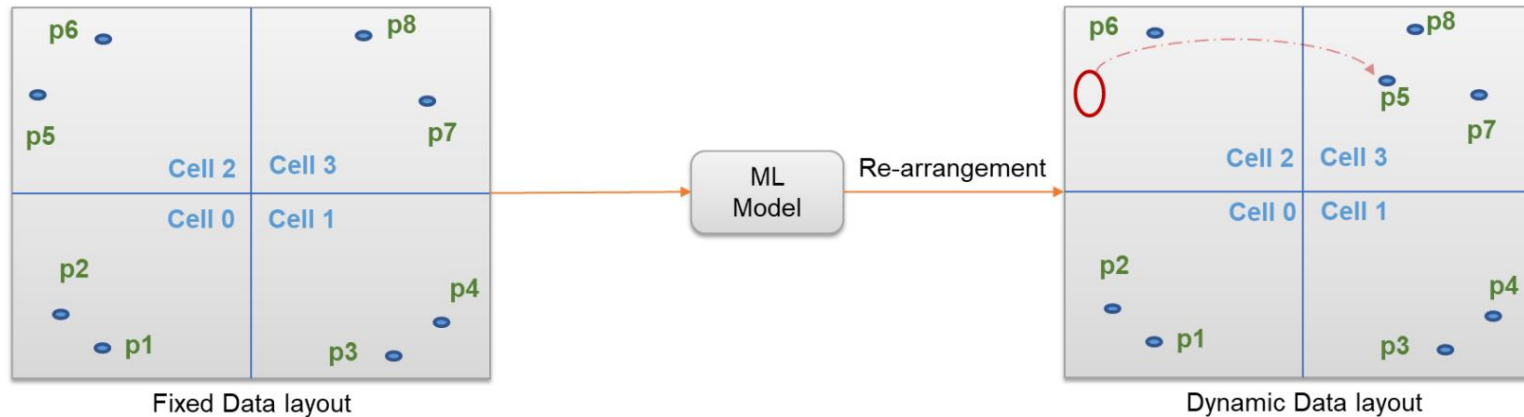
Dimension 1: Immutable vs. Mutable Indexes

- Given a Learned Index, can it support updates?
- Why are updates challenging?
 - The learned index takes significant time to train.
 - New data will require retraining because it changes the old learned ordering of the data.
- Classify learned indexes based on the ability to support updates:
- **Immutable Learned Index:**
 - Does not support inserts, updates, or deletes
- **Mutable Learned Index:**
 - Supports inserts, updates, or deletes

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Dimension 2: Fixed vs. Dynamic Data Layouts



- Mainly, there are two types of data layouts for learned indexes:
 - **Fixed Data Layout:**
 - The layout of the data and the structure of the index are fixed before building the learned index.
 - **Dynamic Data Layout:**
 - The layout of the data is arranged and can be modified by the ML models while building and updating the learned index.

Outline

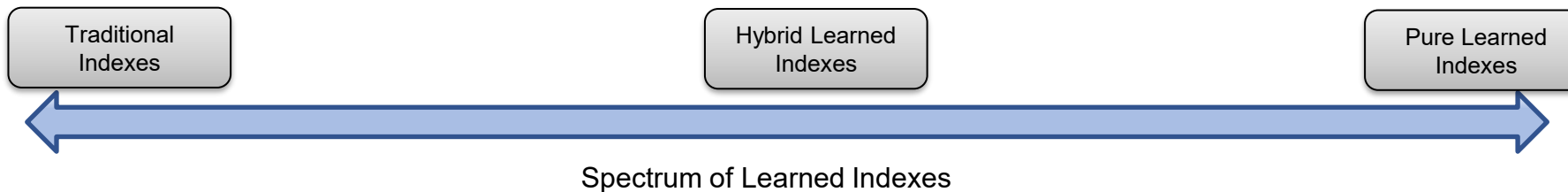
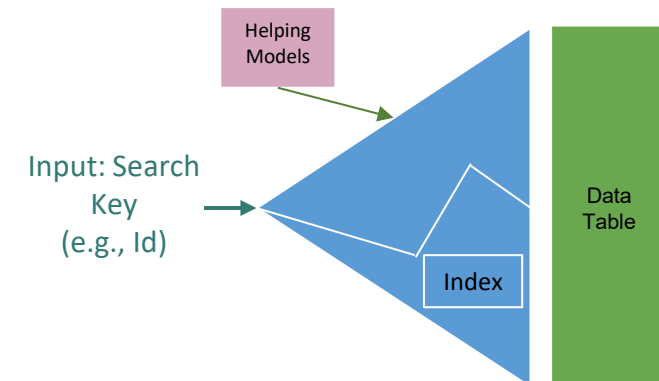
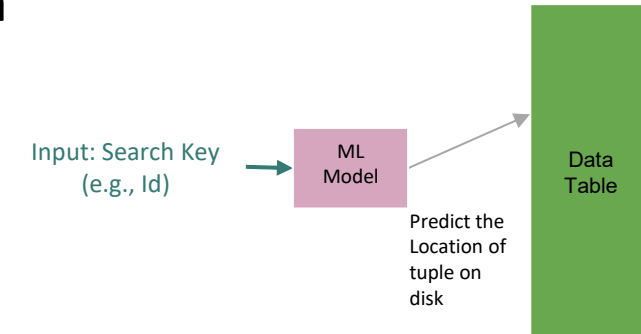
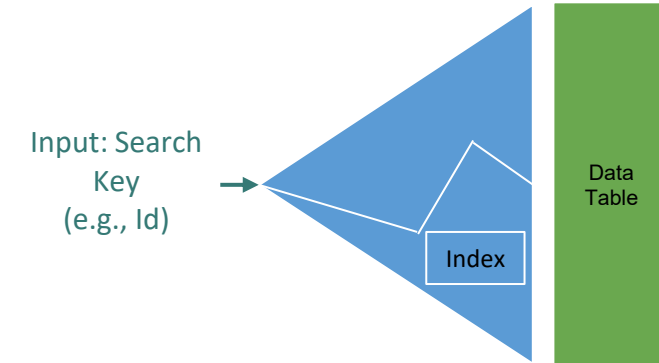
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Dimension 3: Pure vs. Hybrid Indexes

- **Pure Learned Indexes:**
 - Designed without leveraging a traditional index structure (e.g., a B-tree or an R-tree)
 - Replace a traditional index structure
- **Hybrid Learned Indexes:**
 - Combine a traditional index structure with ML models to build an ML-enhanced index structure

Dimension 3: Pure vs. Hybrid Indexes (Cont'd)

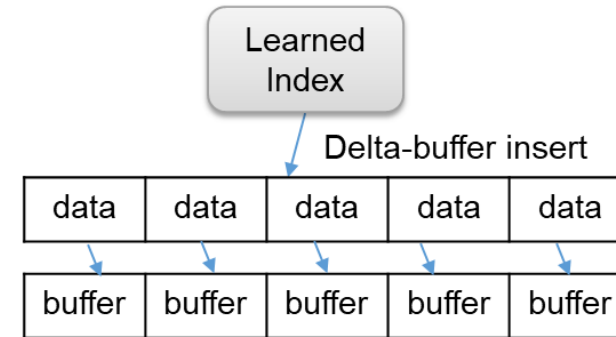
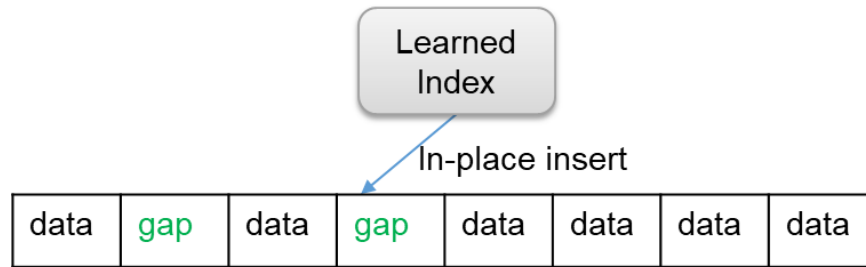
- **Traditional Indexes:**
 - Theoretical guarantee on performance
 - Well studied and successfully integrated in real systems
- **Pure Learned Indexes:**
 - Learn search-key distribution with some error correction mechanism
 - Better performance with less space requirement
- **Hybrid Learned Indexes:**
 - Optimizing traditional indexes with helping (e.g., ML) models



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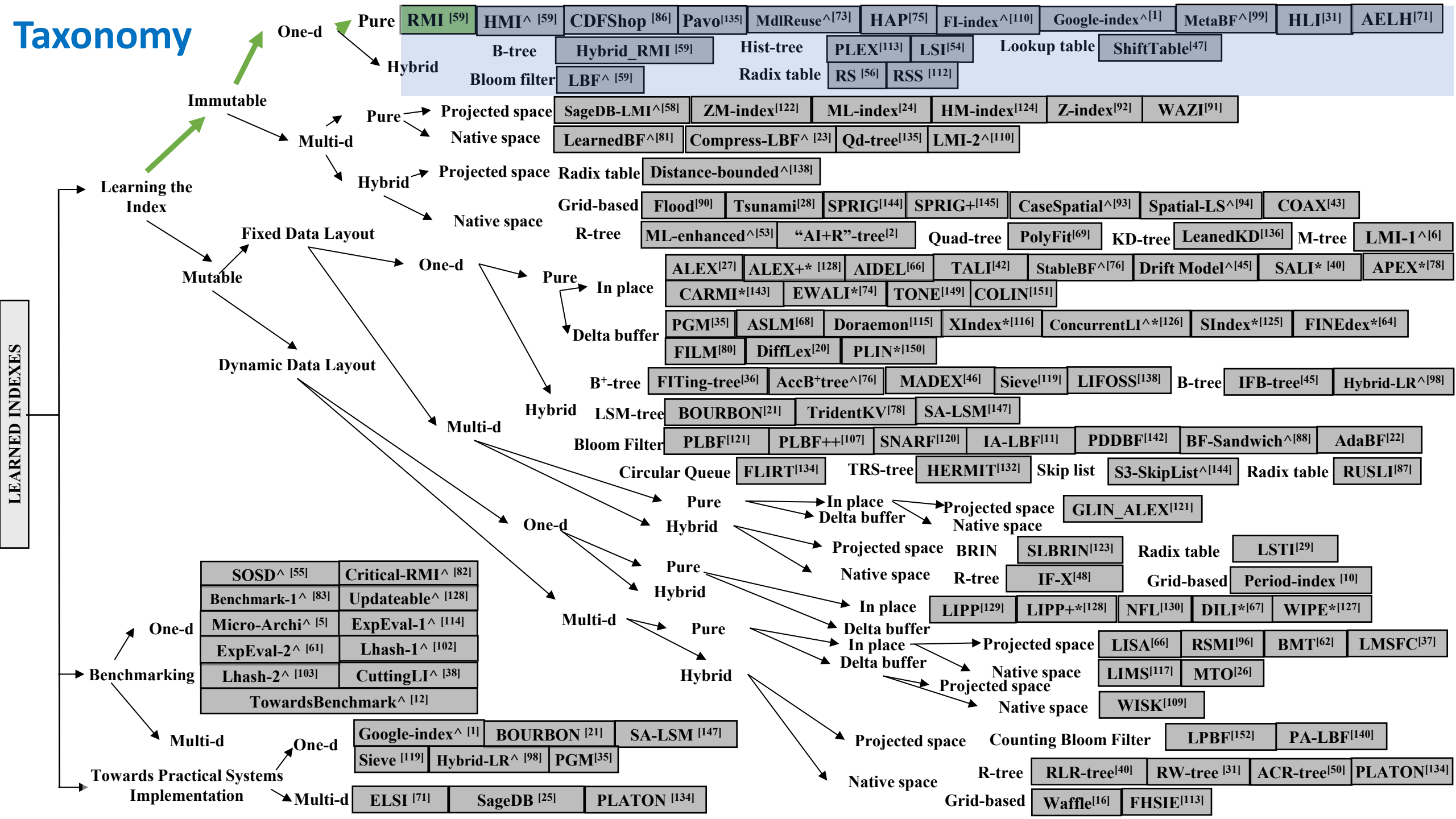
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Dimension 4: In-place vs. Delta Buffer Insertion Strategies



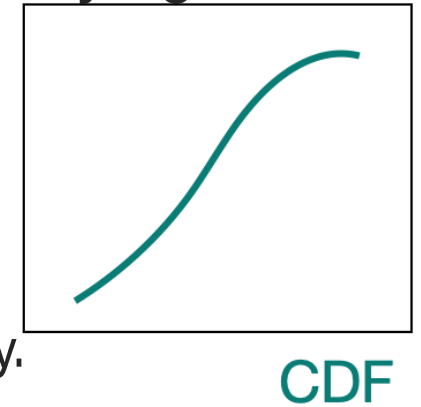
- **In-place:**
 - Use ML models to find the position where the new data item should be inserted
 - Reserve some gaps in the index so that insert operations can be accommodated immediately
- **Delta Buffer:**
 - Employ fixed-size buffers to hold the new data items
 - Synchronize with the index structure periodically (e.g., during ML model re-training)

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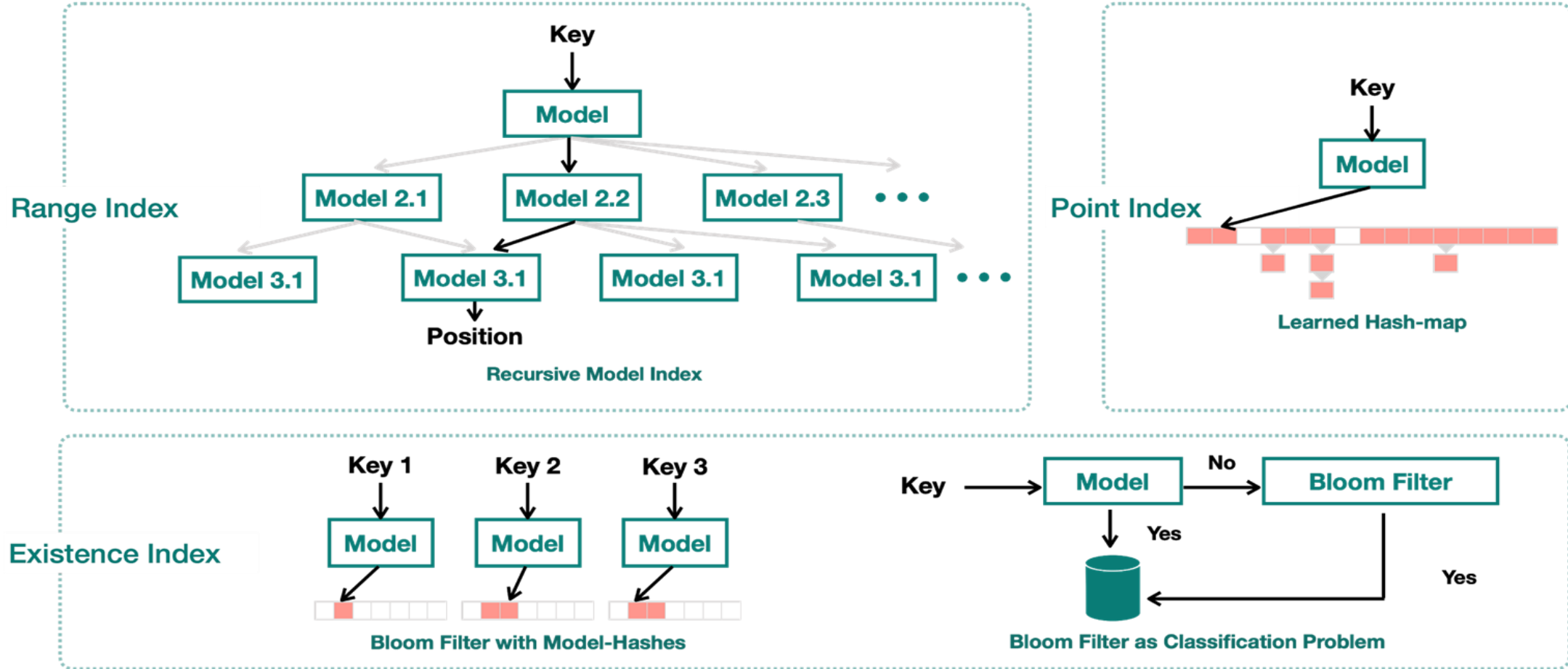


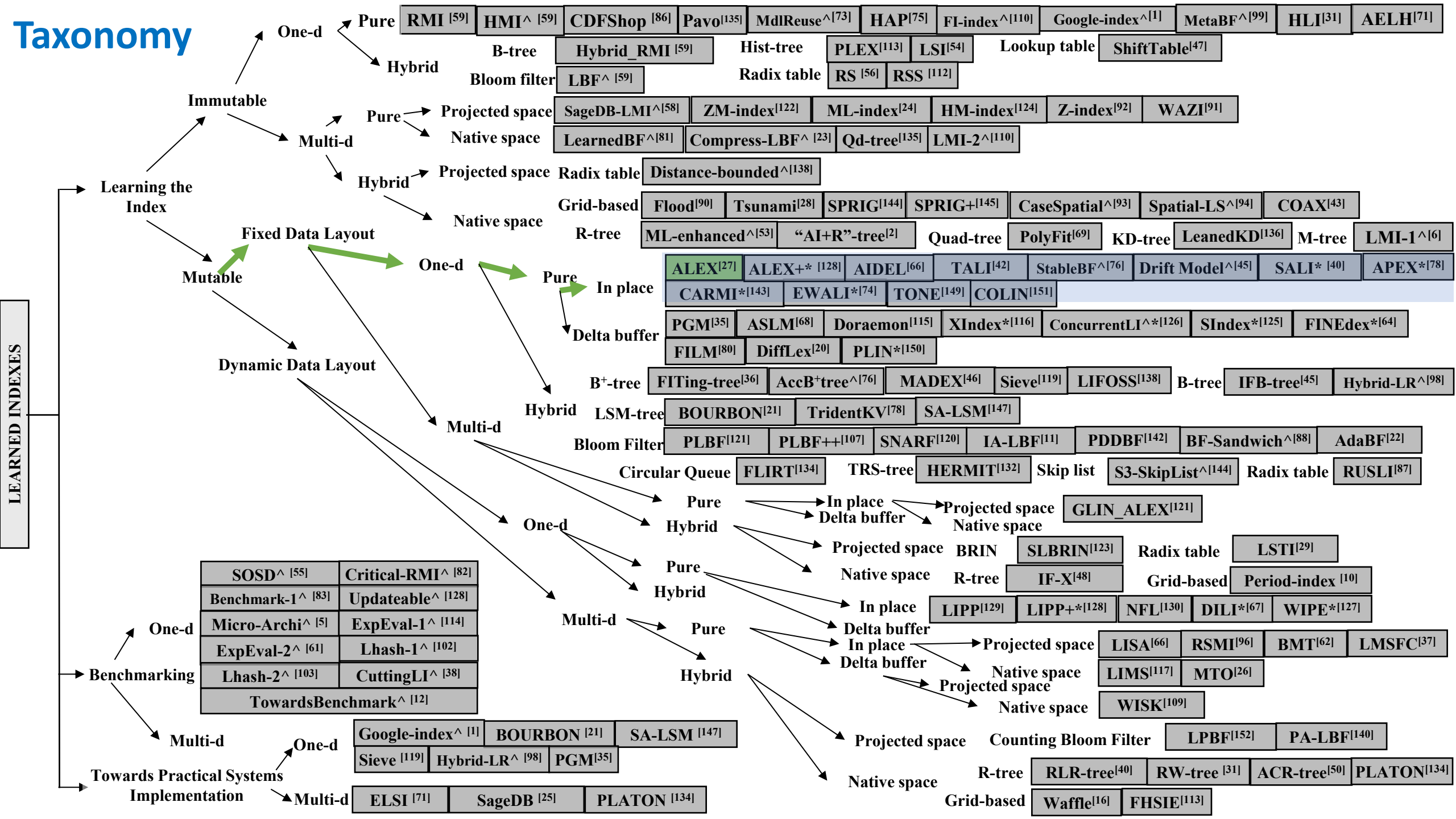
Recursive Model Index (RMI) [SIGMOD'18]

- An **Immutable Pure** Learned Index
- Approximate the Cumulative Distribution Function (CDF) of the underlying (sorted) data
- A multi-stage ML model
- Combine simpler ML models
 - The first stage model will make an initial prediction of the CDF for a specific key.
 - The next stage models will be selected to refine this initial prediction.
- Proposed Learned Index Structures: Range Index, Point Index, and Existence Index

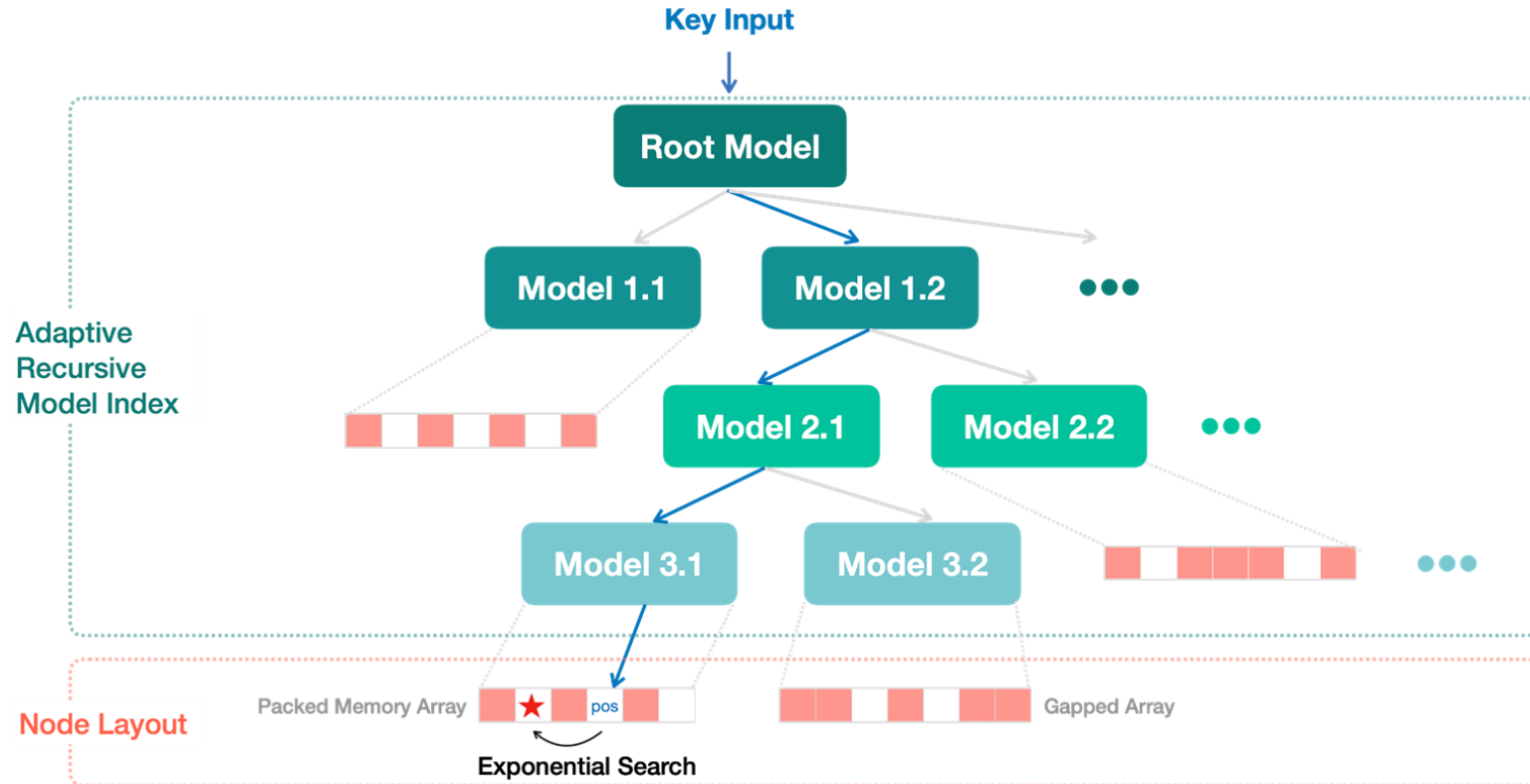


Recursive Model Index (RMI) [SIGMOD'18]





- A **Mutable Pure** learned index with **Fixed Data Layout**
- Adopts an **In-place** insertion strategy
- Adaptive RMI as Model Hierarchy
- Linear Regression Model as Node
- Gapped Array or Packed Memory Array as Node Layout



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Learned Multi-dimensional Indexes: Background

- One of the earliest distribution-aware spatial indexes can be found in:
 - Babu, G. Phanendra. "Self-organizing neural networks for spatial data." Pattern Recognition Letters 18, no. 2 (1997): 133-142.
- Can ML models replace and act in place of a multi-dimensional index?
 - Yes, ML models can act in place of a multi-dimensional index, e.g., R-Tree

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Learned Multi-dimensional Indexes: Challenges

- **Sorting/ordering of multi-dimensional data:**
 - No obvious sort order for multi-dimensional data
- **Error correction mechanism in case of misprediction:**
 - Difficult to define an error correction mechanism in case of mispredictions
- **Choice of the ML model:**
 - Which ML models to choose?
- **Layout of the data:**
 - How to arrange the data?
 - Affect range query time and model accuracy

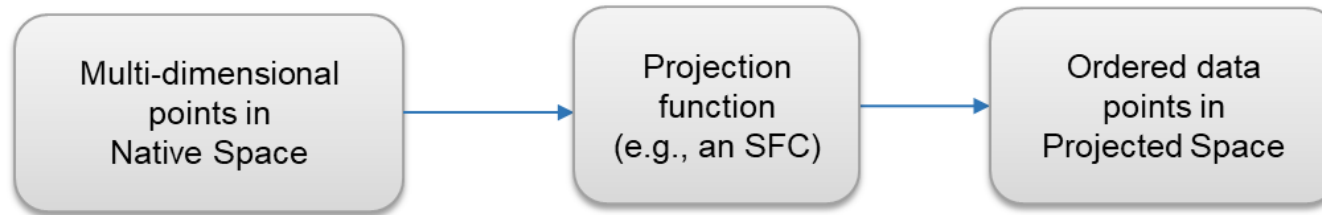
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Learned Multi-dimensional Indexes: Projected vs. Native Space

- **Projected Space:**

- Multi-dimensional data is projected or linearized into a projected space using a projection function or a Space Filling Curve (e.g., Z-order, Hilbert-curve).



- **Native Space:**

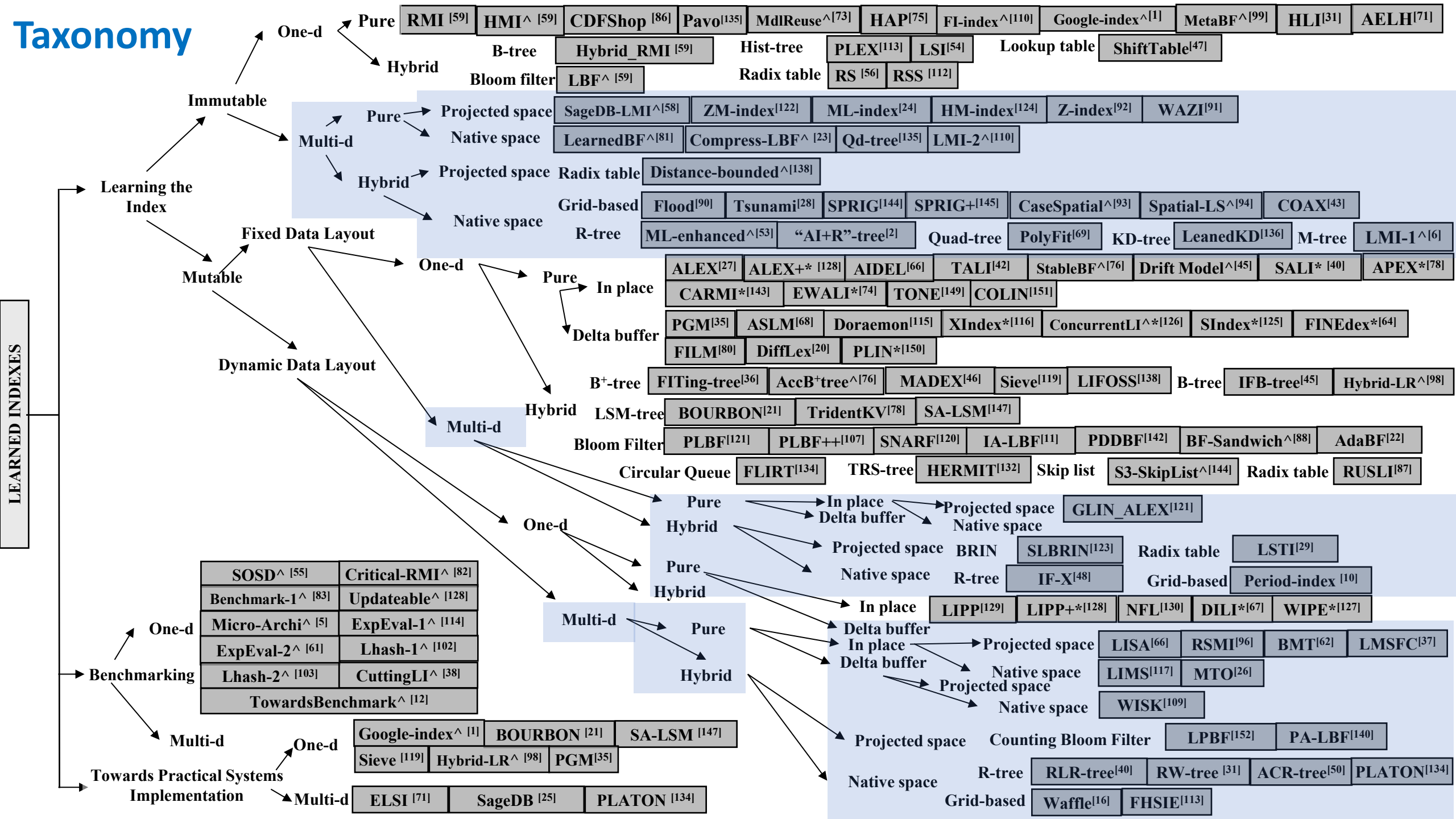
- ML models are trained on the original representation of the multi-dimensional data.

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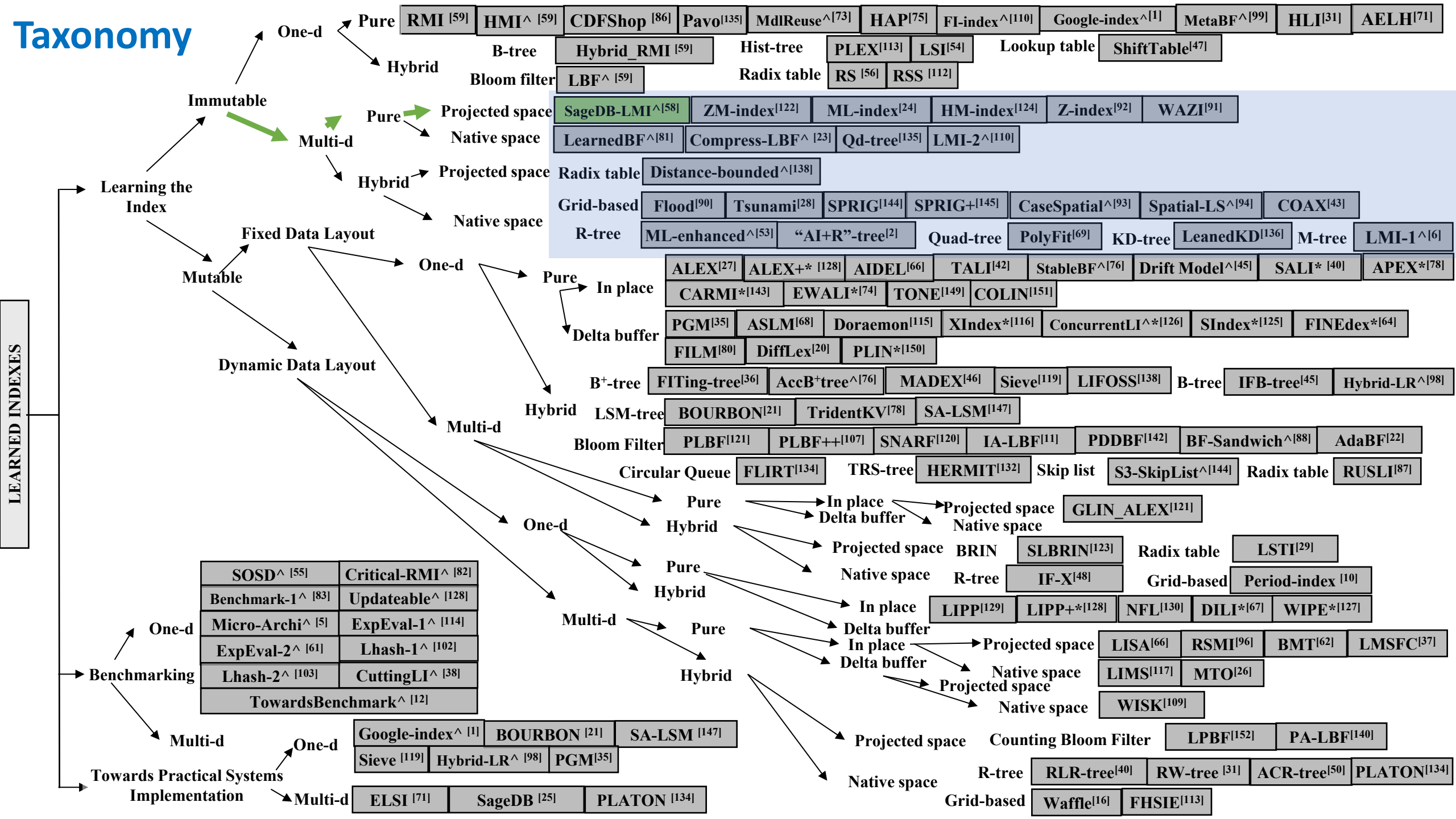
Learned Multi-dimensional Indexes: Approaches

- The concept of learned one-dimensional indexes has been extended in multi-dimensional spaces mainly by four approaches:
 - **Approach 1:**
 - Augment traditional multi-dimensional index with ML models
 - Keep the traditional index
 - **Approach 2:**
 - Project multi-dimensional data into one-dimensional space and use a learned one-dimensional index
 - Replace the traditional index
 - **Approach 3:**
 - Apply a one-dimensional learned index in the native multi-dimensional space without using a projection function
 - Replace the traditional index
 - **Approach 4:**
 - Design learned multi-dimensional indexes from scratch
 - Create a data layout that is suitable for learning
 - Replace the traditional index



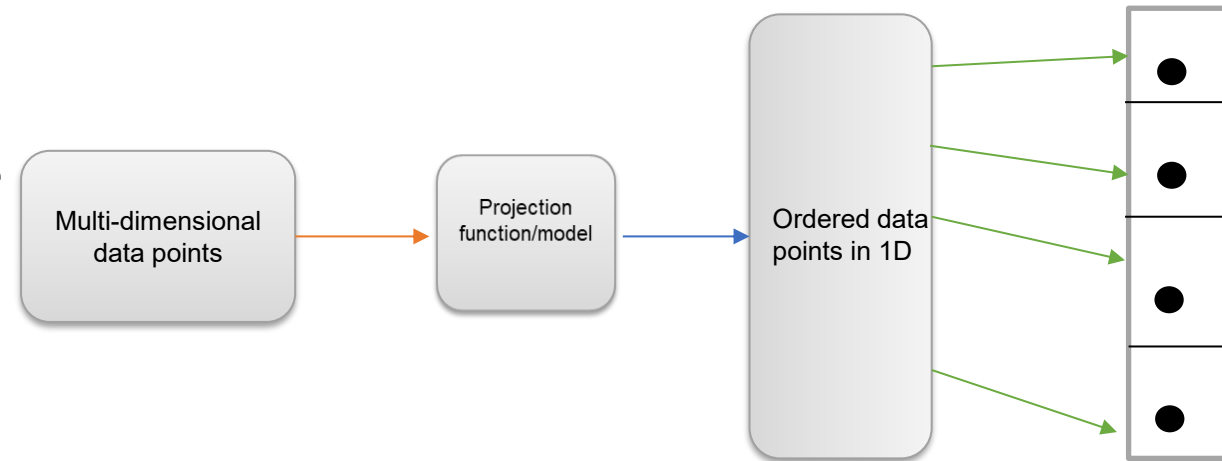
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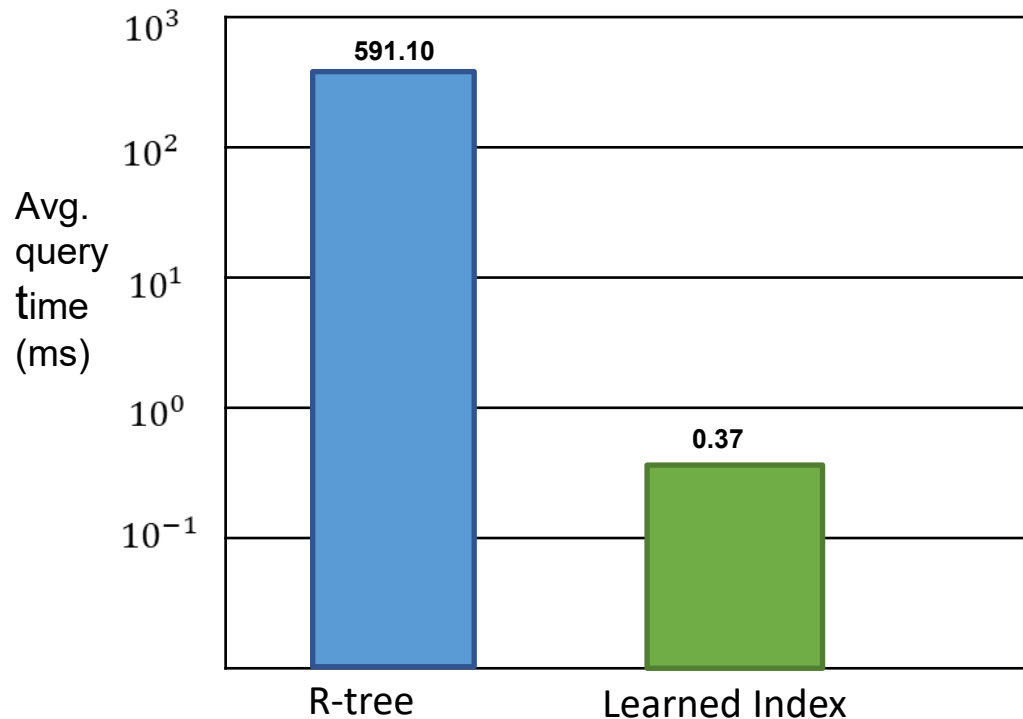
Sagedb: A Learned Database System [CIDR'19]

- Propose a learned multi-dimensional index by applying RMI in a projected space
- **Step-1: Project the multi-dimensional data points into one-dimensional space**
 - Consecutively sorting and partitioning the points along a sequence of dimensions (e.g., first x-dimension then y-dimension) into uniformly-sized cells
 - Produce a layout that is efficient to compute and easily learnable
 - Comparing with Z-order which is difficult to learn
- **Step-2: Use a trained CDF model (e.g., RMI) to predict the physical location of the point**

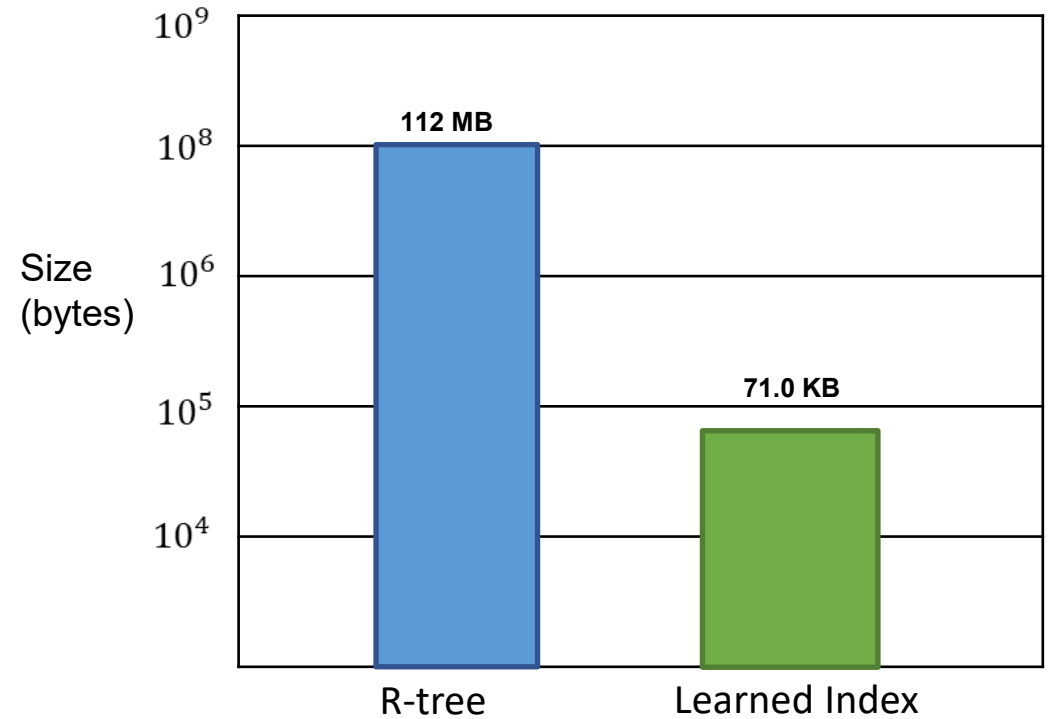


Sagedb: A Learned Database System [CIDR'19] (Cont'd)

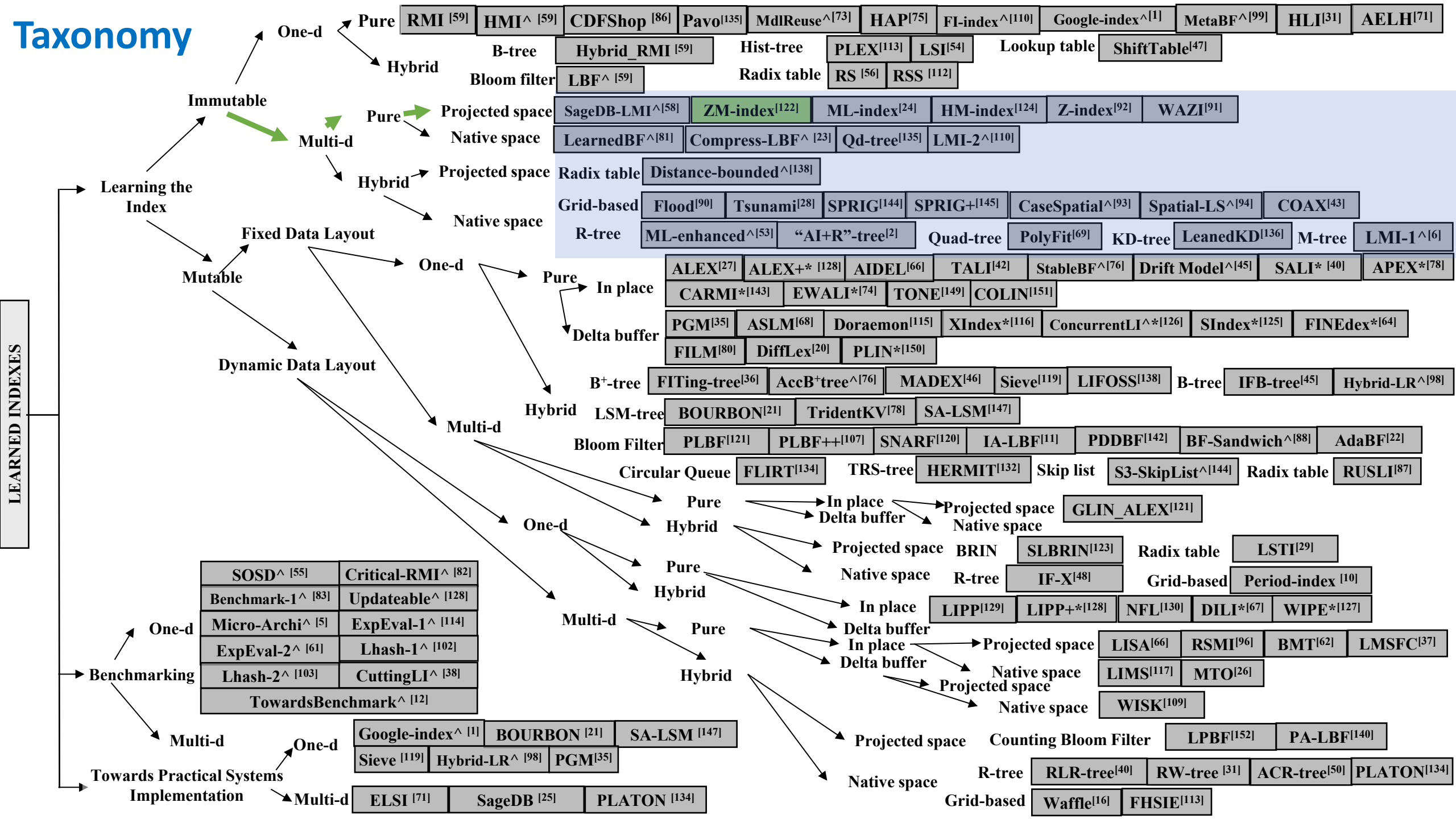
- Initial Result:
 - R-Tree vs. Learned Multi-dimensional Index on TPC-H data



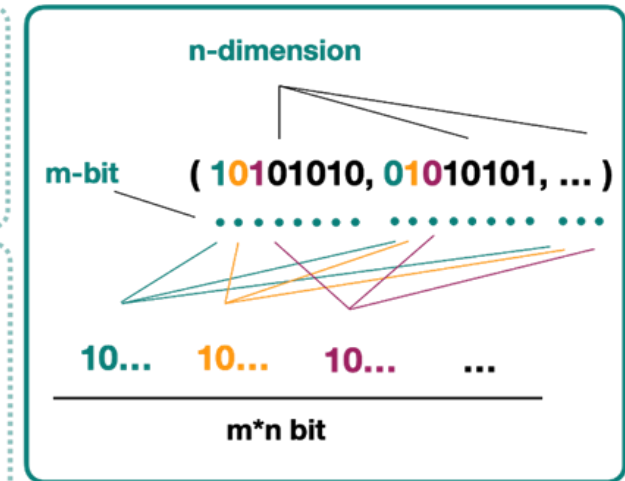
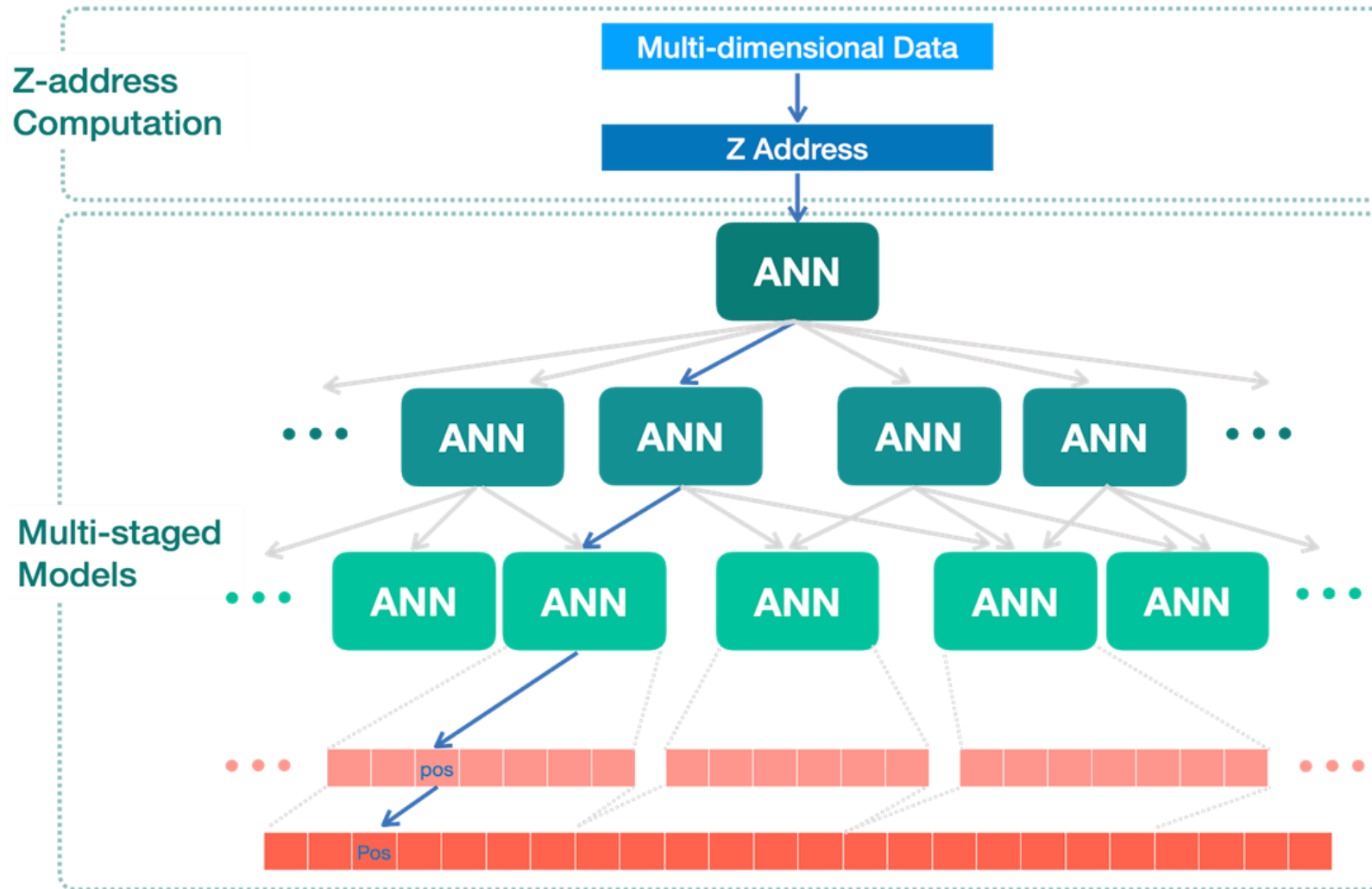
Result on average query time



Result on index size

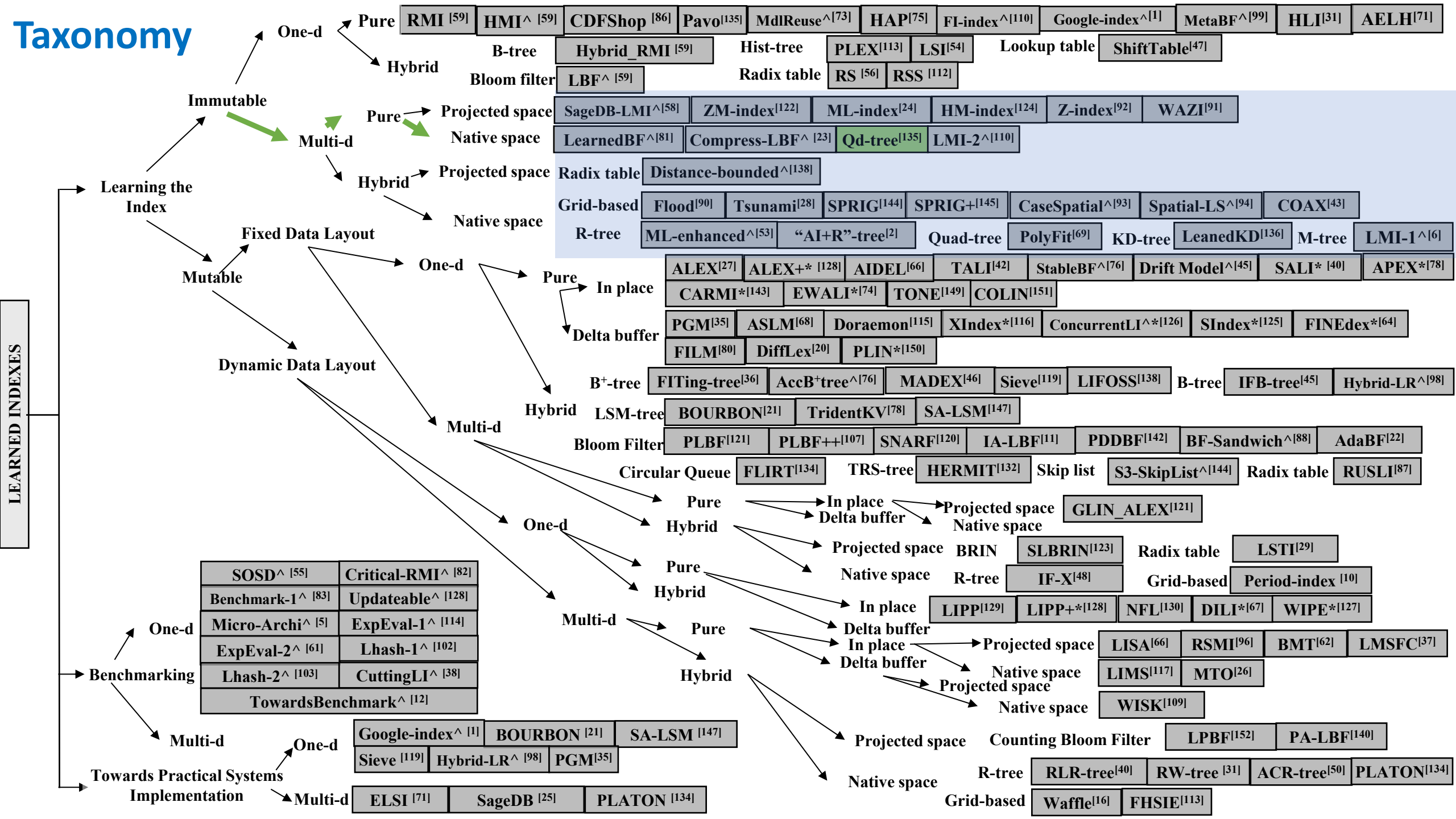


Z-Order Model Index (ZM-Index) [MDM'19]



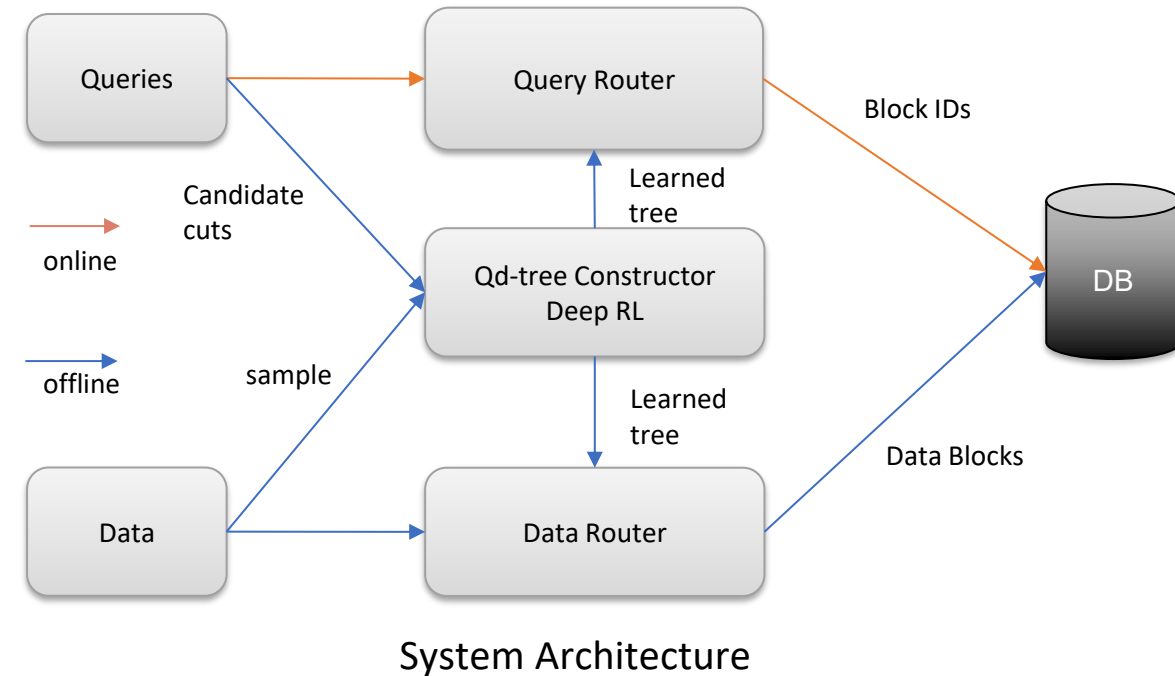
Z-address computation of n-d vector

- **ZM-Index:**
 - Use the Z-order to map the multi-dimensional values into the one-dimensional space
 - Use a multi-staged model (e.g., RMI) for learning
 - Support point and range queries



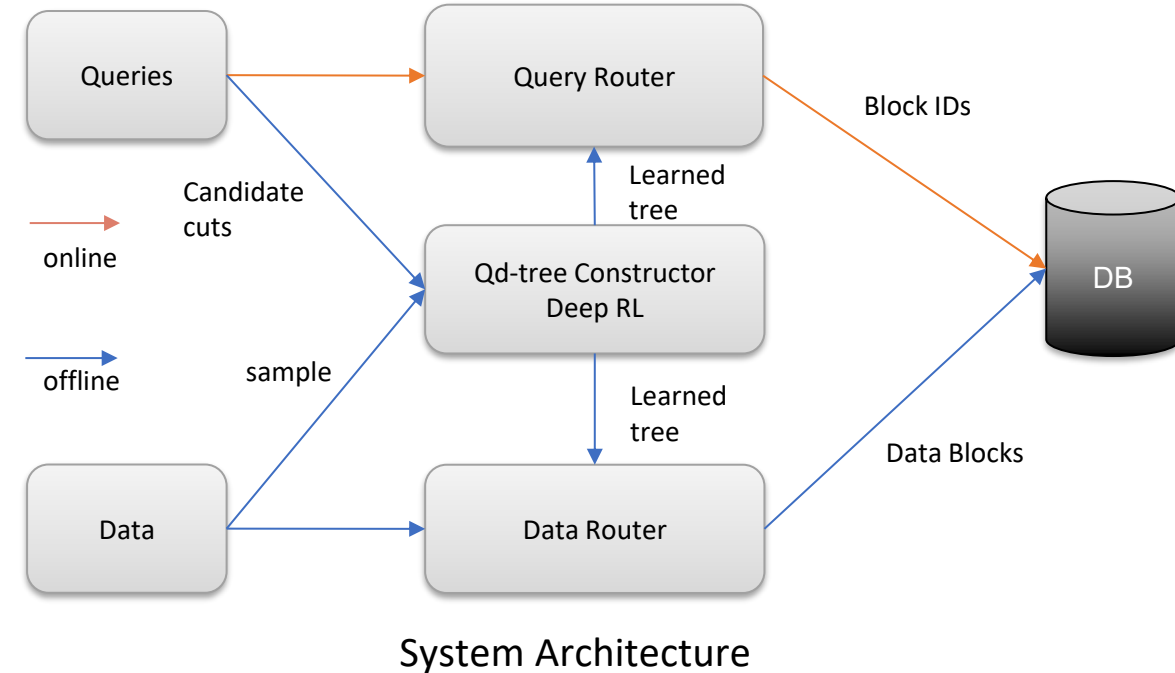
Query-data Routing Tree (Qd-tree) [SIGMOD'20]

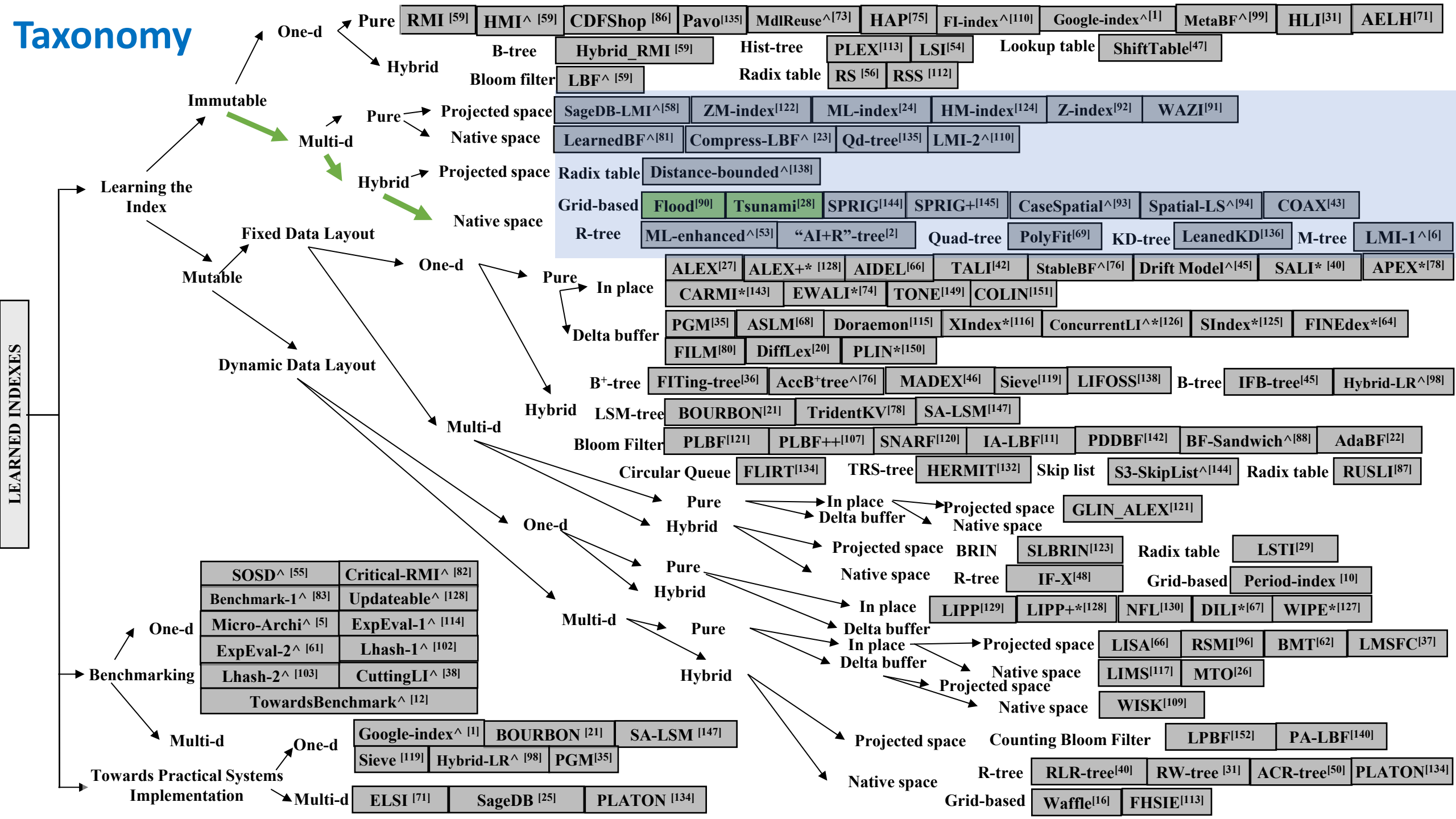
- For disk-based systems, an important performance metric is:
 - The number of data blocks accessed by a query.
- Minimize the number of blocks/records accessed by a given workload
 - Generate block-level data layouts with high I/O performance
- QD-trees are neural network-generated decision trees that
 - Recursively partition the data space into smaller subspaces.
- Use deep Reinforcement Learning (RL) to create Qd-trees
 - Proximal Policy Optimization (PPO)



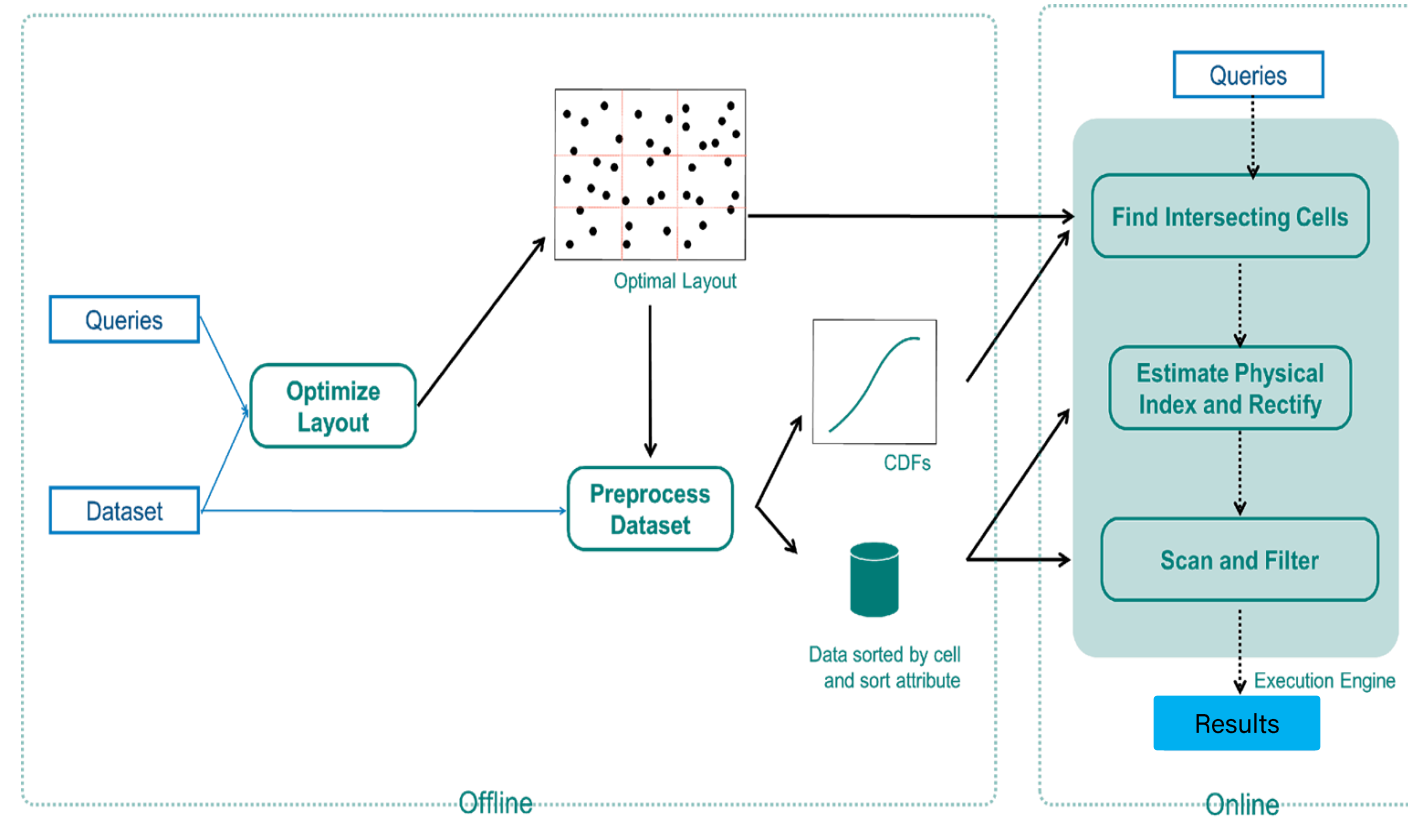
Query-data Routing Tree (Qd-tree) [SIGMOD'20] (Cont'd)

- Experiments over benchmark and real workloads
 - Compared to existing blocking schemes:
 - Provides physical speedups of more than an order of magnitude
- For data skipping based on selectivity:
 - Performs within 2X of the lower bound





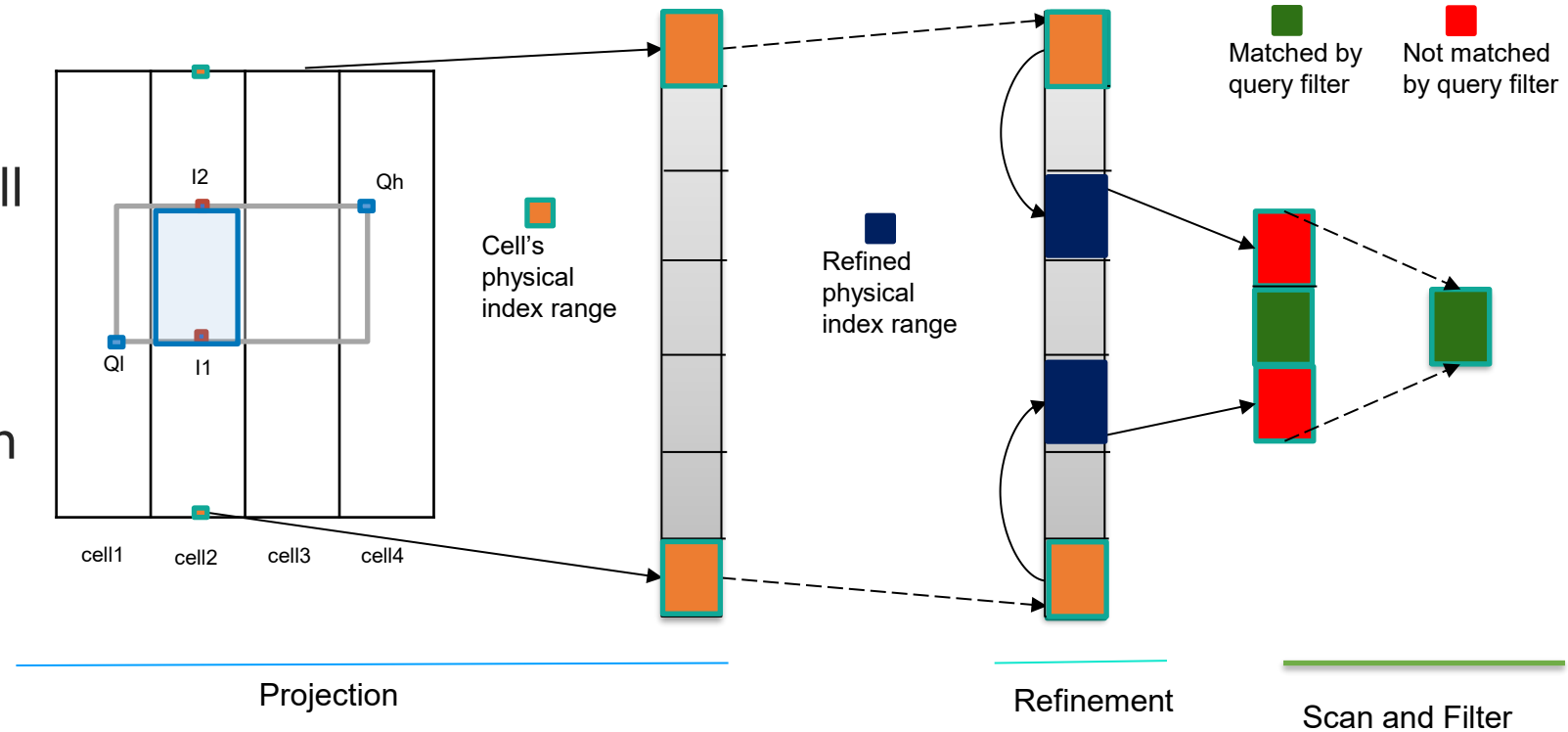
- A read-optimized grid-based learned multi-dimensional index
- Co-optimize the data layout and the index structure
 - For a particular data and query distribution
- Two components:
 - Offline (pre-processing)
 - Pick an optimal layout
 - Create an index based on that layout
 - Online
 - Query execution



Flood's System Architecture

Flood [SIGMOD'20] (Cont'd)

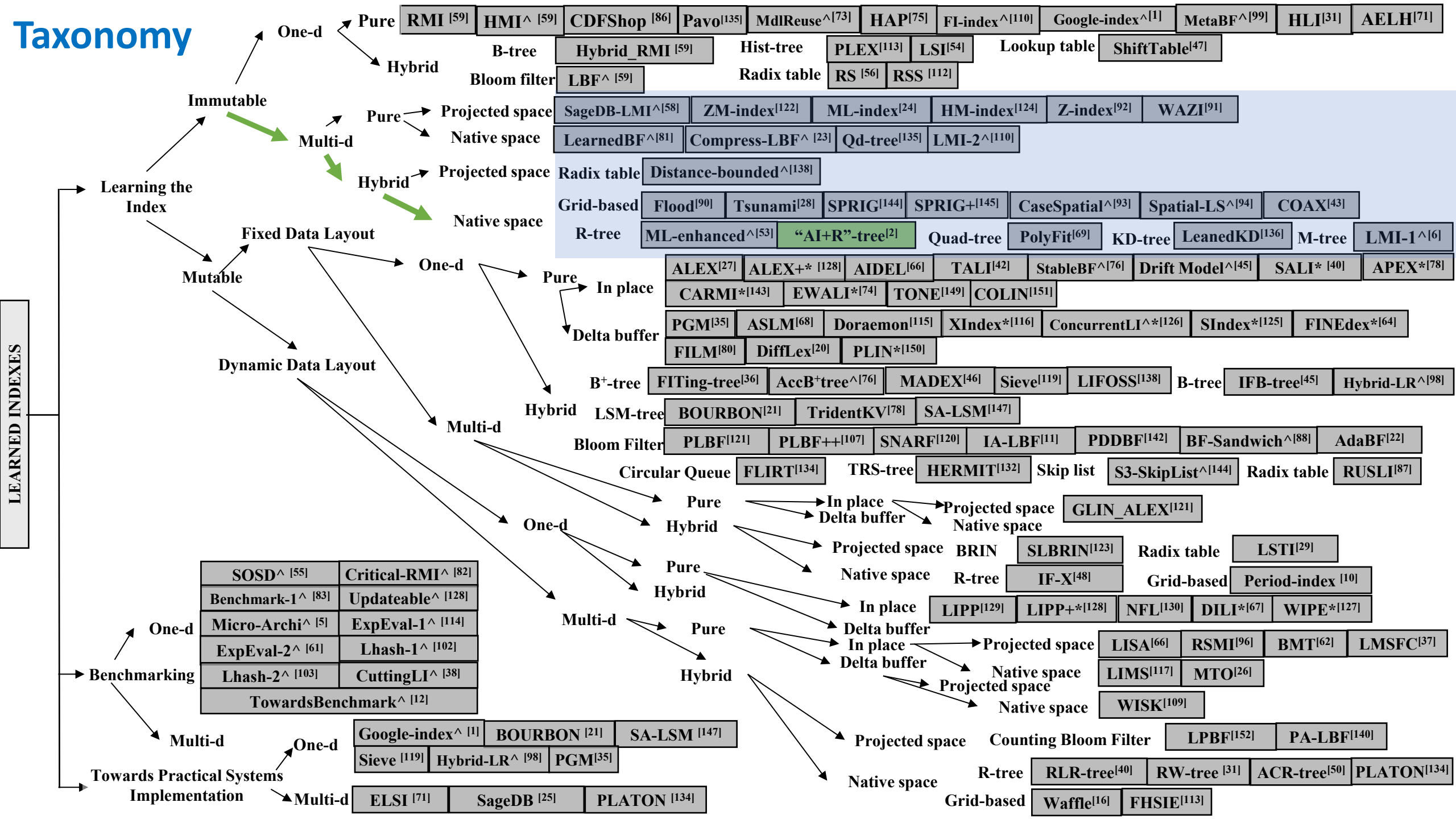
- Projection:
 - Identify the intersecting cells
 - Identify the physical index range in each intersecting cell
- Refinement:
 - Utilize the ordering of points within each cell to refine each physical index range
- Scan and Filter



Flood's Workflow

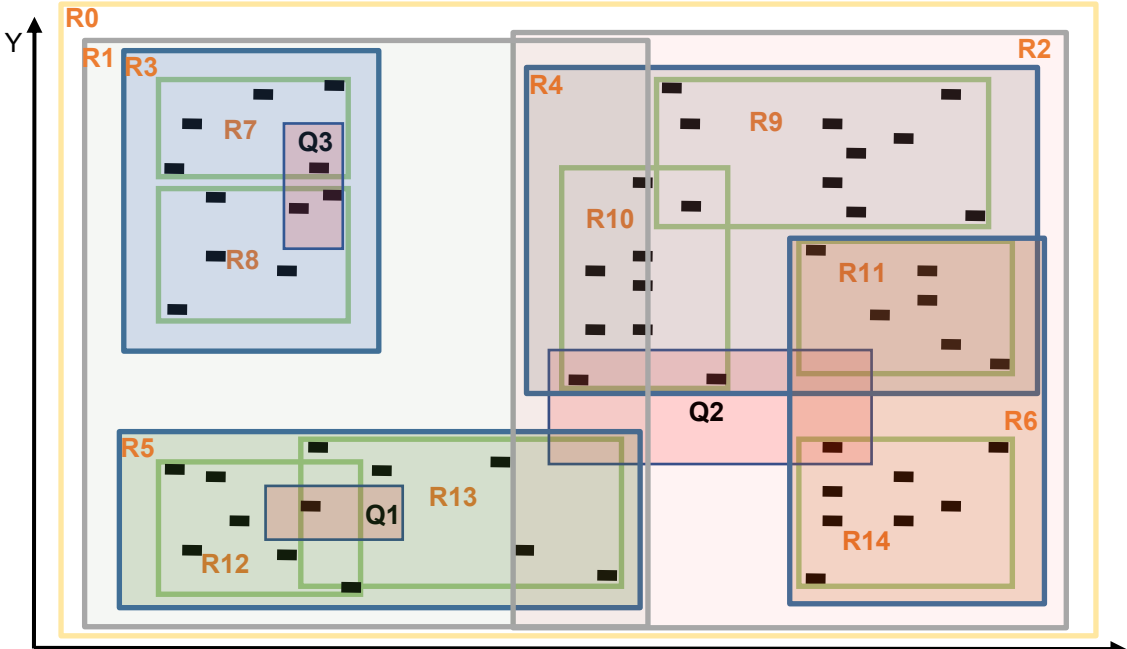
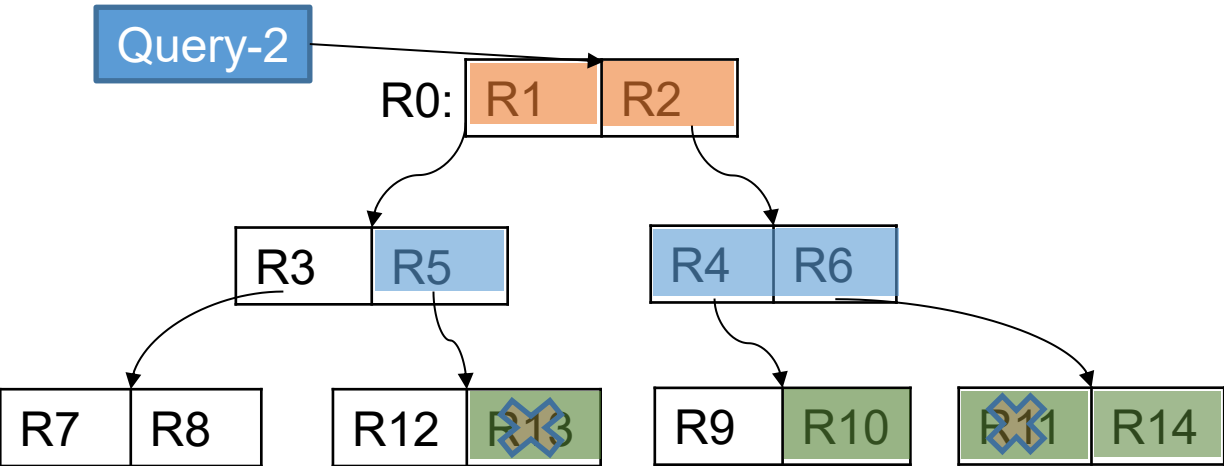
- Experimental Results:
 - Outperform optimally tuned spatial indexes
 - Use only a fraction of the space comparing with traditional indexes
- Limitations:
 - Cannot adapt to skewed query workload
 - If dimensions are correlated,
 - Performance and memory usage are affected

- Extend the idea of Flood to overcome its limitations over skewed query workload and correlated data
- Adaptable to changes in workload
- Scales across data size, query selectivity, and dimensionality
- Outperform Flood in scenarios with skewed query workloads and correlated data



The AI+R-tree [MDM'22]

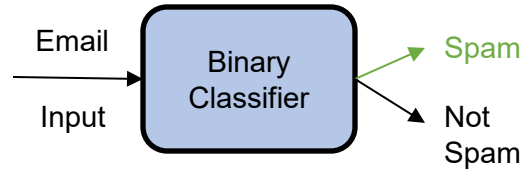
- Problem Formulation:
 - Given a range query $Q(X_{\min}, Y_{\min}, X_{\max}, Y_{\max})$ as input, can we directly predict the true leafNodeIds in the R-Tree that contain the output data objects?



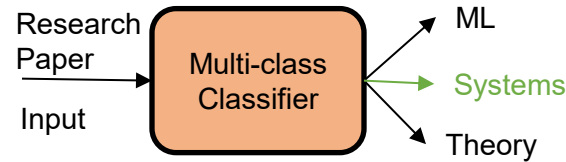
Input	Leaf Node Ids that will be searched by R-Tree	Leaf Node Ids that contains the actual data
Query-1	R12, R13	R13
Query-2	R10, R11, R13, R14	R10, R14

The AI+R-tree [MDM'22] (Cont'd)

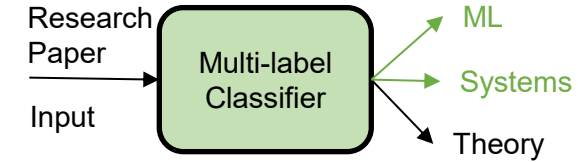
- Three different types of classification tasks:



Total number of classes = 2
For an input, the number of output classes = 1



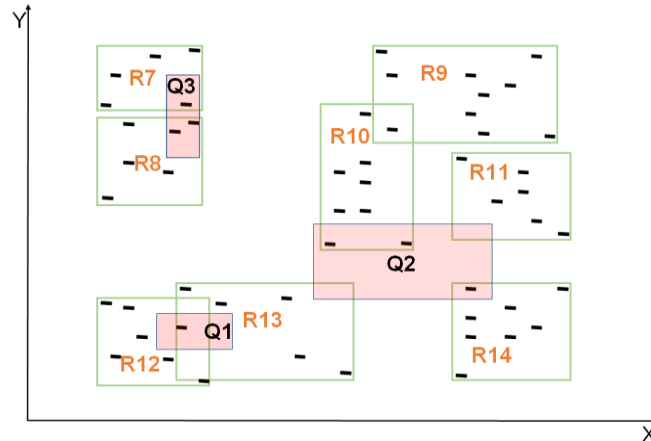
Total number of classes > 2
For an input, the number of output classes = 1



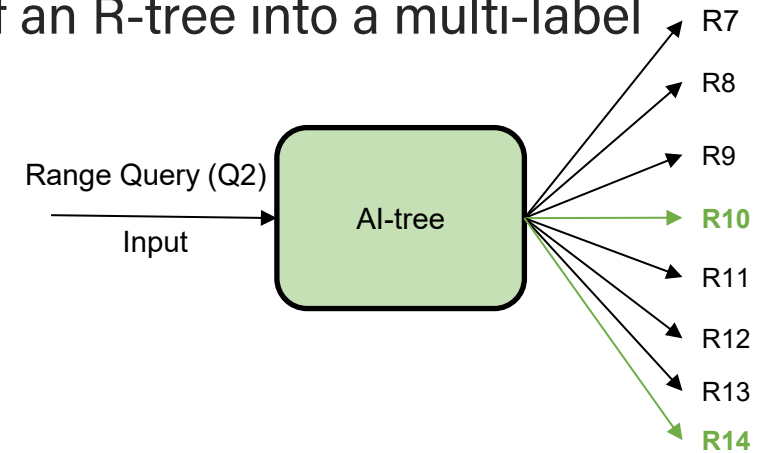
Total number of classes > 2
For an input, the number of output classes ≥ 0

- Proposal:

- The AI-tree which transforms the search operation of an R-tree into a multi-label classification task.



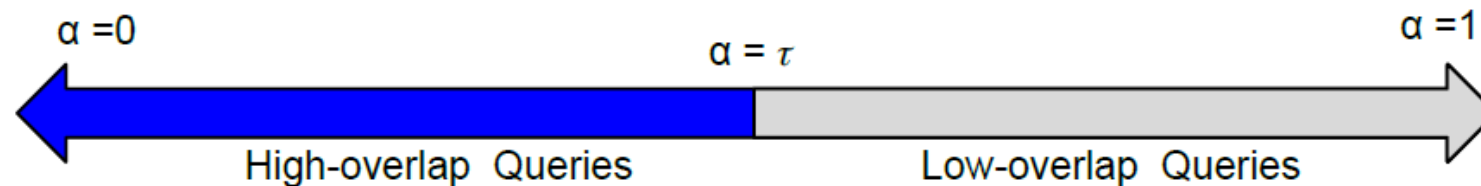
An example of leaf nodes in an R-tree



Total number of classes > 2
For an input, the number of output classes ≥ 0

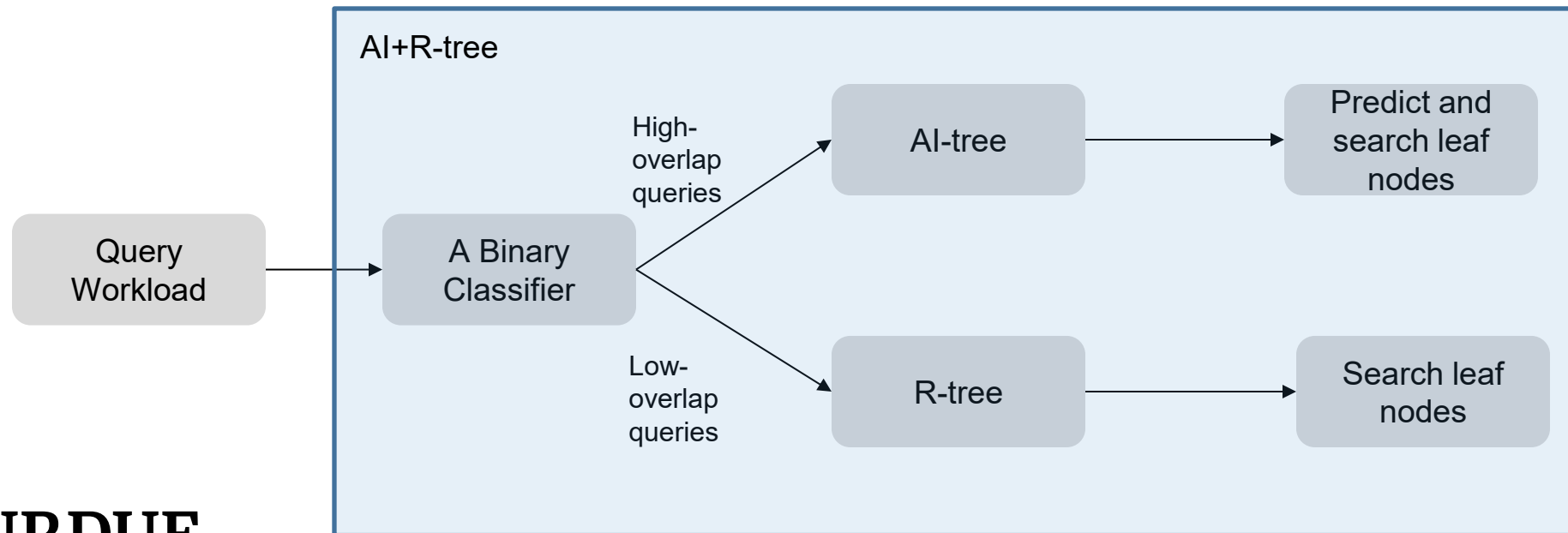
The AI+R-tree [MDM'22] (Cont'd)

- An overlap ratio α quantifies the degree of extraneous leaf node accesses required by a range query.
- For a range query:
 - $\alpha = \frac{\text{Number of true leaf nodes that actually contains the output data objects}}{\text{Number of leaf nodes searched by the R-Tree}}$
 - α is in the range $[0,1]$
- High-overlap Queries:
 - A range query is declared as a “high-overlap query” if the value of α is less than a pre-defined threshold τ .



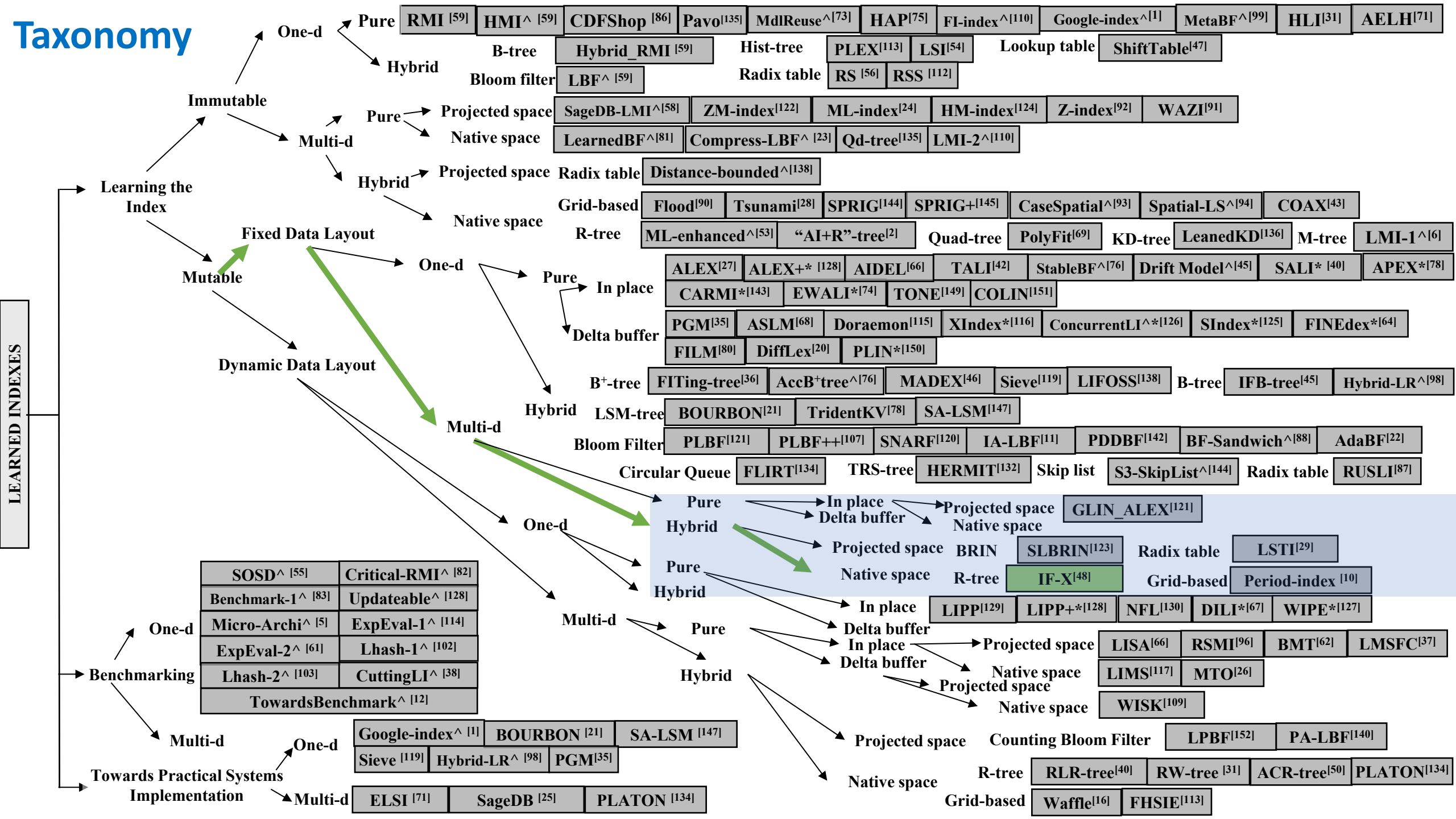
The AI+R-tree [MDM'22] (Cont'd)

- Query Processing:
 - Leveraging a binary classifier to automatically differentiate between high- and low-overlap queries
 - Process the high-overlap queries using the AI-tree
 - Process the low-overlap queries using the R-tree
 - Adopting a hybrid approach, namely the AI+R-tree, to process both types of range queries efficiently



Outline

- ✓ • Introduction and Historical Background
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 - ✓ • Motivation and Challenges
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 - ✓ • Approaches
 - ✓ • Immutable Indexes
 - ➡ • Mutable Indexes with Fixed Data Layouts
 - Mutable Indexes with Dynamic Data Layouts
 - Open Challenges for Future Research



Interpolation Friendly (IF) Indexes: IF-X [EDBT'20]

- IF-X:

- X can be any multi-dimensional index.
- Why Linear Interpolation?
 - Complex models have a higher capacity to fit the CDF.
 - But complex models
 - Require more parameters
 - Slower to compute
- Linear interpolation is:
 - Simpler
 - Computationally inexpensive
 - Can eliminate expensive training process
- Query execution time can be reduced by up to 60%.
- Memory footprint can be reduced by over 90%

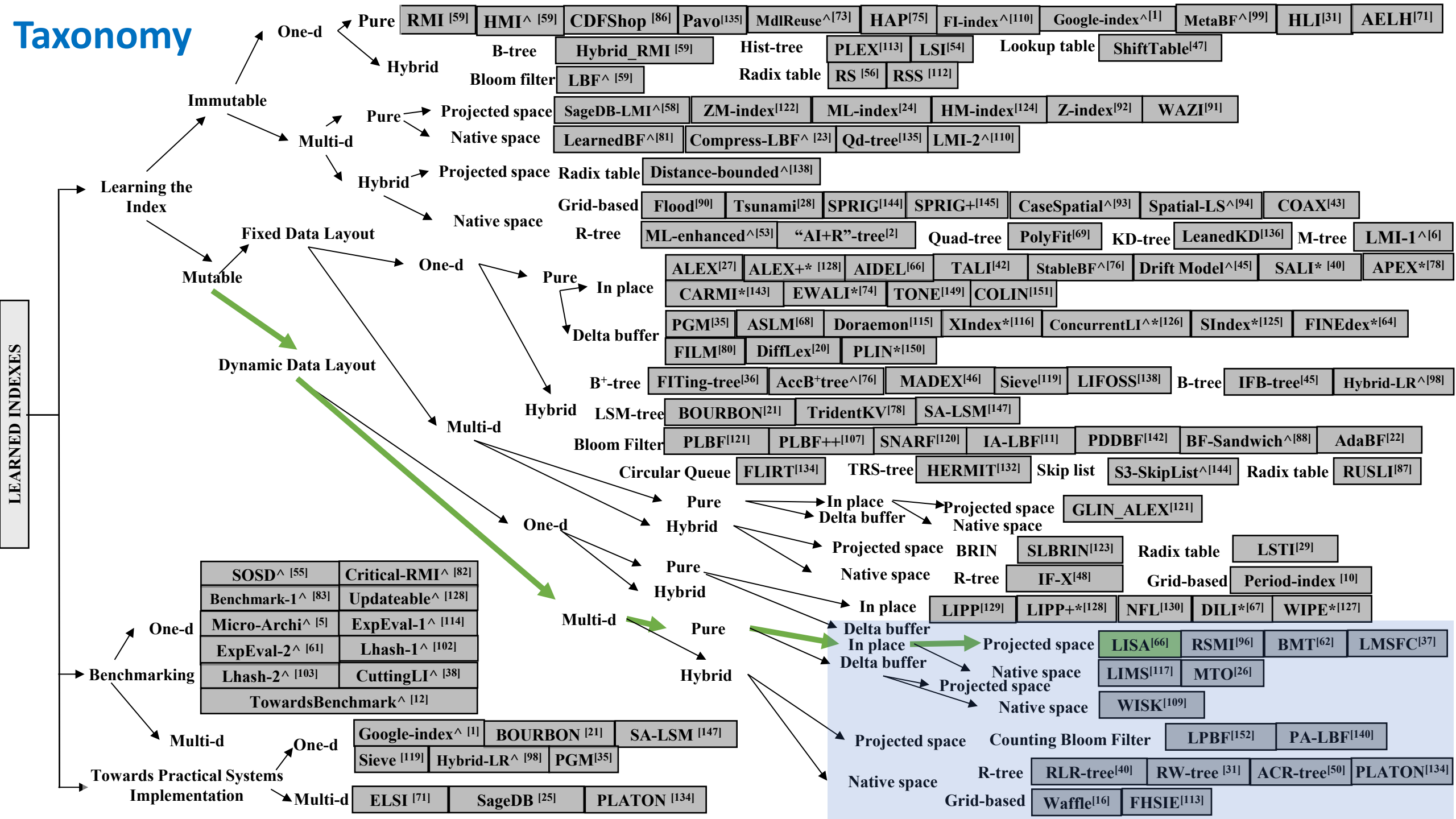
No of records (#)	Max error (delta)	Prediction axis (pDim)	Slope (A)	Base (C)	Records sorted by pDim
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- Leaf Node Layout:

- IF-X indexes sort the records in each leaf node
 - Based on the best order, the interpolation error is minimized.
- Store all required information in the header of the leaf node
 - No additional computation is needed
- The leaf node structure:
 - pDim: most predictable dimension which is used as the storage order

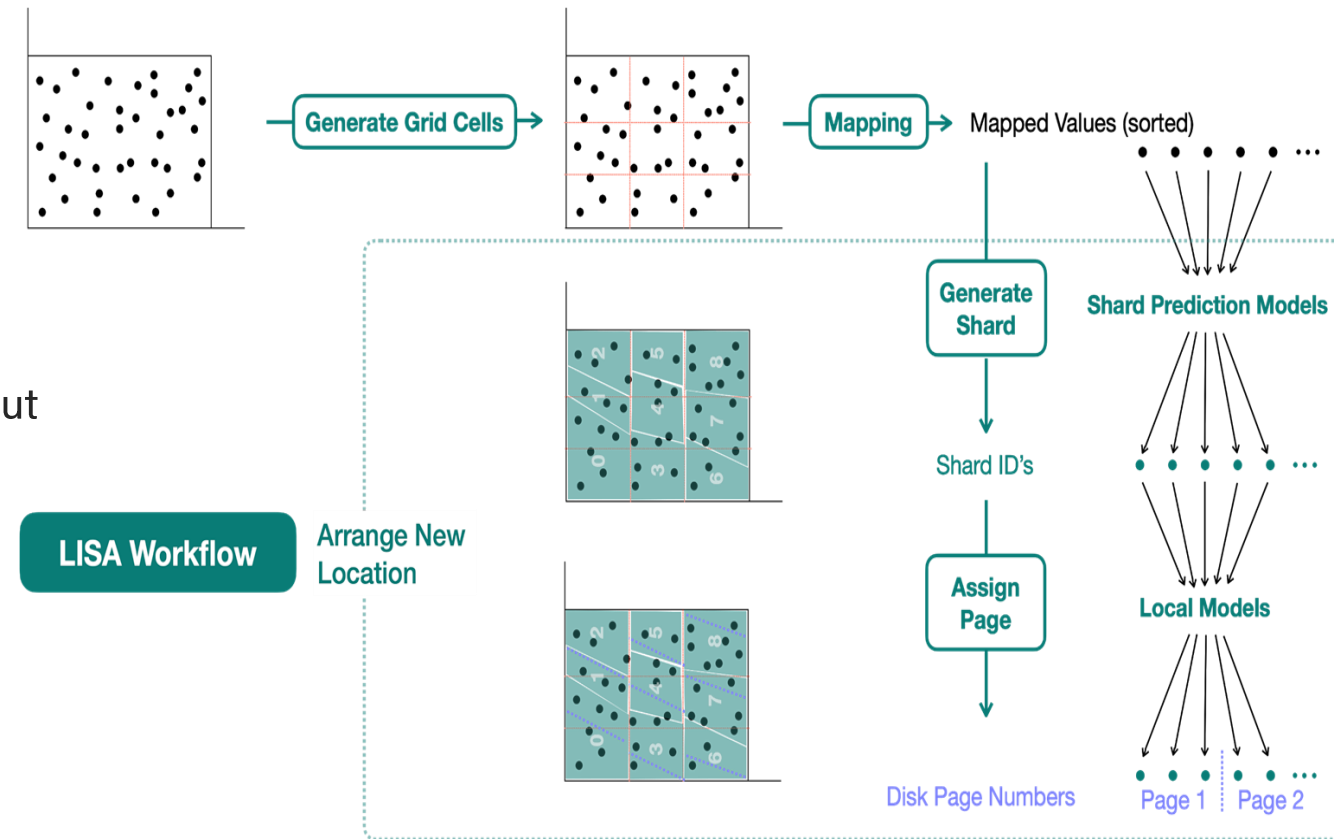
Outline

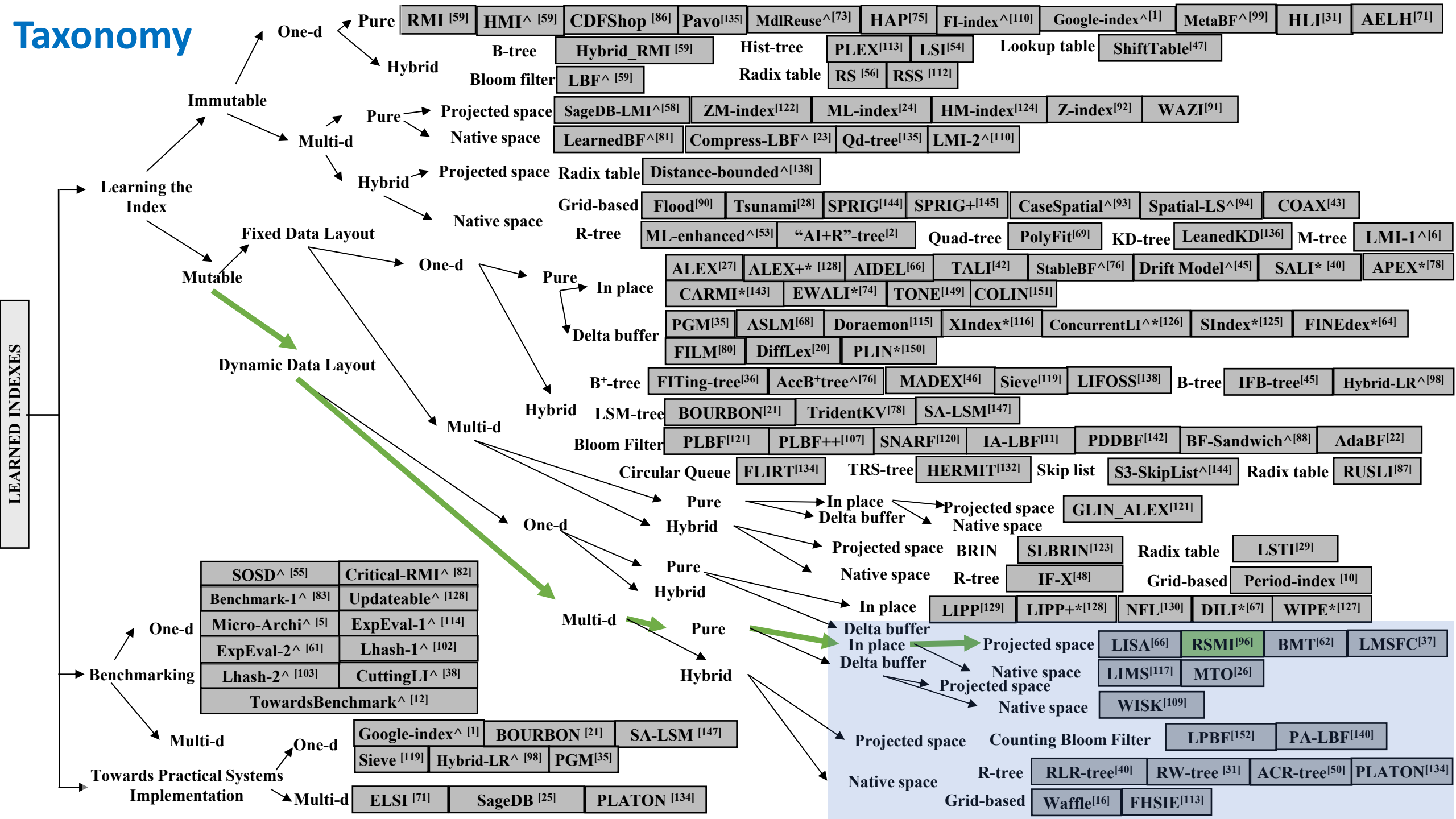
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Learned Index for Spatial Data (LISA) [SIGMOD'20]

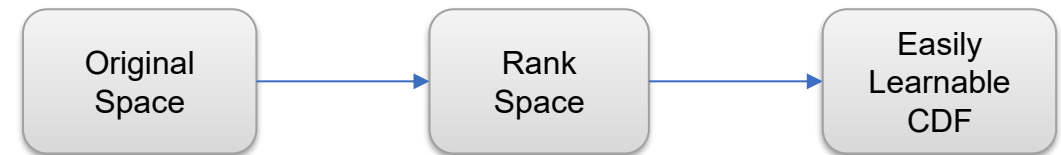
- A Mutable disk-based learned multi-dimensional index for spatial queries.
- **Mapping function:**
 - $M(\text{spatial keys}) \rightarrow 1\text{D mapped values}$
- **Learned Shard Prediction Function:**
 - $SP(\text{mapped value}) \rightarrow \text{Shard Id}$
 - Use ML models to generate searchable data layout in disk pages for arbitrary spatial dataset
- **Local models:**
 - Assign pages for all shards and perform intra-shard operations
- Outperforms traditional spatial indexes for range and KNN queries:
 - Memory consumption
 - IO cost





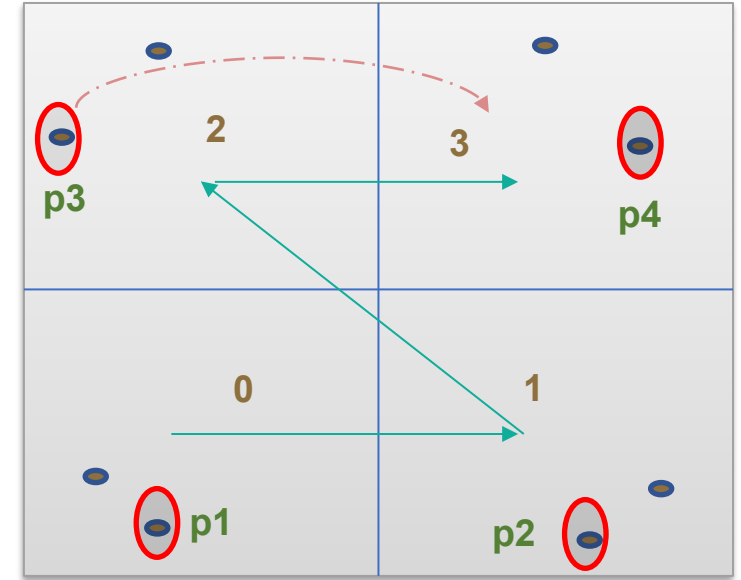
Recursive Spatial Model Index (RSMI) [VLDB'20]

- Selecting grid resolution for Z-order for learned multi-dimensional index (e.g. ZM-Index) is challenging:
 - Large cells
 - More false positives due to many points per cell
 - Small cells
 - Hard to learn due to uneven gaps in Cumulative Distribution Function (CDF)
- Spatial index based on ordering the data points by a rank space-based transformation
 - Simplify the indexing functions to be learned
 - $M(\text{search keys}) \rightarrow \text{disk block Ids (location)}$

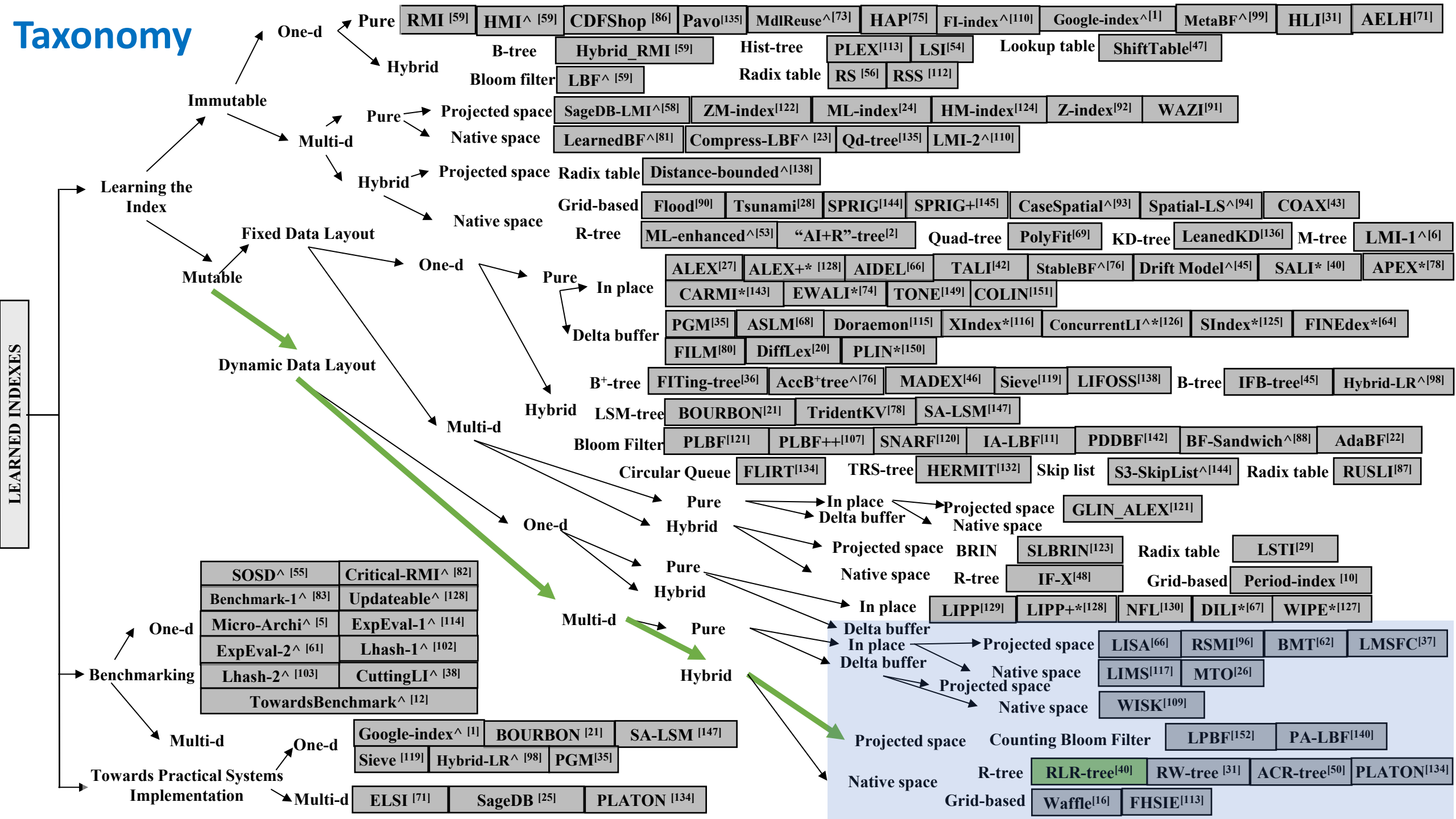


Recursive Spatial Model Index (RSMI) [VLDB'20] (Cont'd)

- Recursively partition a dataset
- Partitioning is learned over the distribution of data
- **Steps:**
 - Initially distribute the data into equal sized partitions
 - Use a Space Filling Curve (SFC) to assign Ids to partitions
 - Learn the partition Ids using a model $M_{0,0}$
 - Rearrange the data based on the prediction of $M_{0,0}$
 - Recursively repartition
 - Until each partition can be learned with a simple model



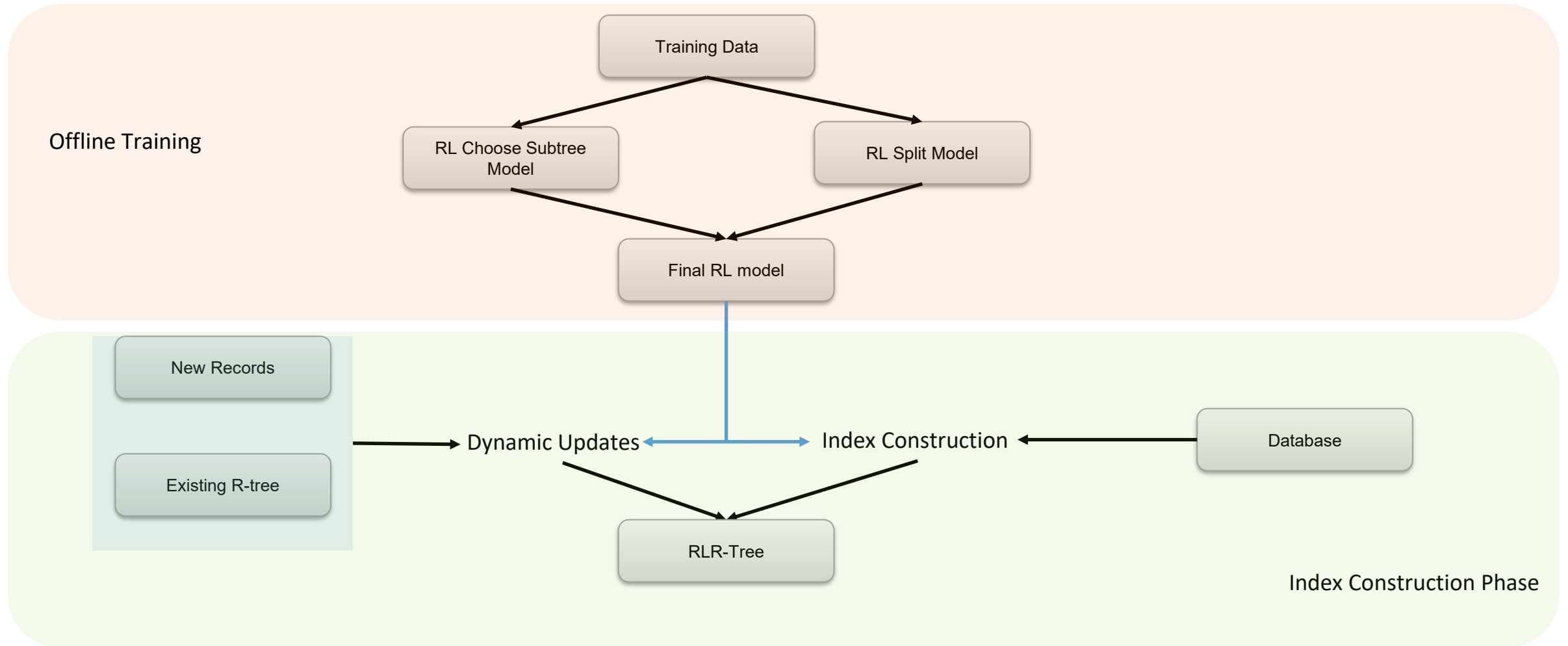
Point	p1	p2	p3	p4
Initial partition Id	0	1	2	3
Model predicted Id	0	1	3	3
Learned partition Id	0	1	3	3



A Reinforcement Learning Based R-Tree (RLR-tree) [SIGMOD'23]

- Formulate the ChooseSubtree and Split operations of a traditional R-tree as a Markov Decision Process (MDP)
- Do not modify existing query processing algorithms
- **State Space:**
 - A tree node with the following features: area increase, perimeter increase, overlap increase, and occupancy rate.
- **Action Space:**
 - Given a current state, the RL agent picks a child node into which the new entry will be inserted.
- **Transition Function:**
 - The agent moves to a child node.
 - If the child node is a leaf node, the agent reaches a terminal state.
- **Reward Function:**
 - Use a reference tree with a pre-defined ChooseSubtree and Split strategy
 - Difference between the query processing cost of the RLR-tree and the reference R-tree
- The RLR-tree adopts a Deep Q-Network learning method for training the agent.

A Reinforcement Learning Based R-Tree (RLR-tree) [SIGMOD'23] (Cont'd)



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Learned Multi-dimensional Indexes: Summary

Properties of the Immutable Learned Multi-dimensional Indexes

Index	Pure Learned	Hybrid Learned	Data Space	Point Query	Range Query	kNN Query	Join Query
SageDB-LMI [101]	✓	×	Projected	Exact	Exact	×	×
ZM-index [197]	✓	×	Projected	Exact	Exact	×	×
ML-index [45]	✓	×	Projected	Exact	Exact	Exact	×
HM-index [199]	✓	×	Projected	Exact	Exact	×	×
Z-index [153]	✓	×	Projected	Exact	Exact	×	×
WAZI [152]	✓	×	Projected	Exact	Exact	×	×
LearnedBF [136]	✓	×	Native	—	—	—	—
CompressLBF [44]	✓	×	Native	—	—	—	—
Qd-tree [214]	✓	×	Native	Exact	Exact	×	×
LMI-2 [183]	✓	×	Native	Approx.	Approx.	Approx.	×
Distance-bounded [219]	×	Radix table	Projected	Approx.	Approx.	×	Approx.
Flood [148]	×	Grid	Native	Exact	Exact	×	×
Tsunami [49]	×	Grid	Native	Exact	Exact	×	×
SPRIG [225]	×	Grid	Native	Exact	Exact	Exact	×
SPRIG+ [226]	×	Grid	Native	Exact	Exact	Exact	×
ML-enhanced [93]	×	R-tree	Native	×	×	Approx.	×
“AI + R”-tree [8]	×	R-tree	Native	Exact	Exact	×	×
CaseSpatial [154]	×	Grid	Native	Exact	Exact	×	×
Spatial-LS [155]	×	Grid	Native	Exact	Exact	×	Exact
COAX [75]	×	Grid	Native	Exact	Exact	×	×
PolyFit [120]	×	Quad-tree	Native	Approx.	Approx.	×	×
LearnedKD [216]	×	KD-tree	Native	×	×	Approx.	×
LMI-1 [13]	×	M-tree	Native	Approx.	Approx.	Approx.	×

Learned Multi-dimensional Indexes: Summary

Properties of the Mutable Learned Multi-dimensional Indexes with Fixed Data Layout

Index	Data Layout	Pure Learned	Hybrid Learned	Data Space	Point Query	Range Query	kNN Query	Join Query
GLIN-ALEX [196]	Fixed	In-place	×	Projected	Exact	Exact	×	×
SLBRIN [198]	Fixed	×	BRIN	Projected	Exact	Exact	Exact	×
LSTI [50]	Fixed	×	Radix Table	Projected	Exact	Exact	Exact	×
IF-X [80]	Fixed	×	R-tree	Native	Exact	Exact	×	×
Period Index [20]	Fixed	×	Grid	Native	Exact	Exact	×	×

Learned Multi-dimensional Indexes: Summary

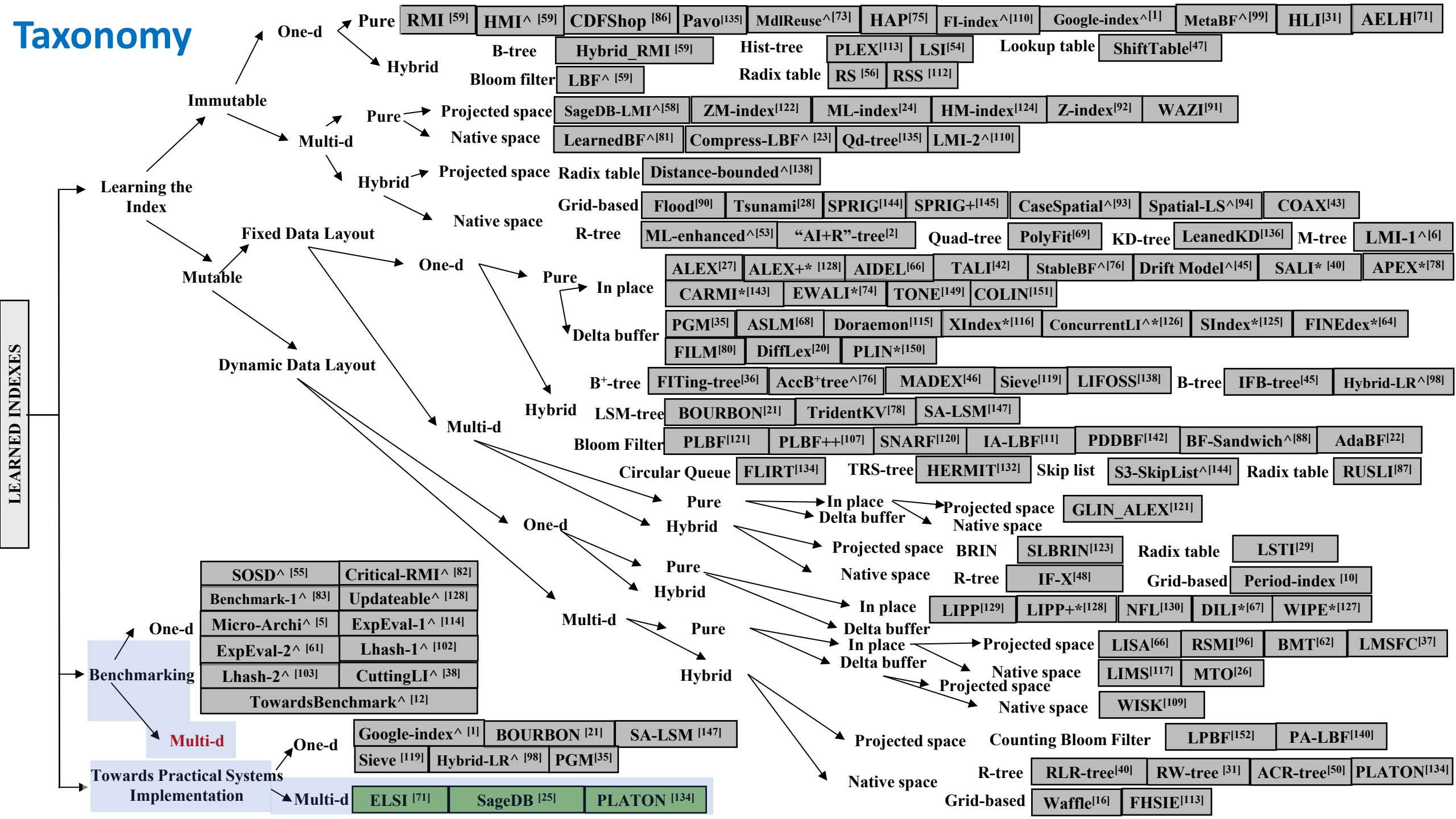
Properties of the Mutable Learned Multi-dimensional Indexes with Dynamic Data Layout

Index	Data Layout	Pure Learned	Hybrid Learned	Data Space	Point Query	Range Query	kNN Query	Join Query
LISA [116]	Dynamic	In-place	×	Projected	Exact	Exact	Exact	×
RSMI [160]	Dynamic	In-place	×	Projected	Exact	Approx.	Approx.	×
BMT [112]	Dynamic	In-place	×	Projected	Exact	Exact	Exact	×
LMSFC [66]	Dynamic	In-place	×	Projected	Exact	Exact	×	×
LIMS [191]	Dynamic	In-place	×	Native	Exact	Exact	Exact	×
MTO [47]	Dynamic	In-place	×	Native	Exact	Exact	×	Exact
WISK [181]	Dynamic	Delta Buffer	×	Native	Exact	Exact	Exact	×
LPBF [234]	Dynamic	×	CBF	Projected	—	—	—	—
PA-LBF [221]	Dynamic	×	CBF	Projected	—	—	—	—
RLR-tree [72]	Dynamic	×	R-tree	Native	Exact	Exact	Exact	Exact
RW-tree [52]	Dynamic	×	R-tree	Native	Exact	Exact	Exact	×
ACR-tree [83]	Dynamic	×	R-tree	Native	Exact	Exact	Exact	×
PLATON [213]	Dynamic	×	R-tree	Native	Exact	Exact	Exact	×
Waffle [38]	Dynamic	×	Grid	Native	Exact	Exact	Exact	×
FHSIE [187]	Dynamic	×	Grid	Native	Exact	Exact	Exact	×

Learned Multi-dimensional Indexes: Summary

ML Techniques for Learned Multi-dimensional Indexes

ML Techniques	Index
RadixSpline, Random Forest Regression	LSTI [50]
Lattice Regression Model	LISA [116]
Ploynomial Regression	LIMS [191]
Linear Model	SageDB-LMI [101], Tsunami [49], COAX [75], GLIN_ALEX [196]
Density Model	Z-index [153]
Piecewise Linear Model, RMI	FLOOD [148]
Polynomial Function	PolyFit [120]
Linear Interpolation	IF-X [80]
Spatial Interpolation Function	SPRIG [225], SPRIG+ [226]
Cumulative Histogram	Period-index* [20]
K-means Clustering	LMI-2 [183], FHSIE [187]
Density based Clustering	RW-tree [52]
Random Forest Density Estimation Model	WAZI [152]
Random Forest, Decision Tree	LMSFC [66], "AI+R"-tree [8]
RadixSpline	CaseSpatial [154], Spatial-LS [155], Distance-bounded [219]
Gradient Boosting	LPBF [234], PA-LBF [221]
Neural Network, Linear Model	ZM-index [197]
Neural Network	LearnedKD [216], LMI-1 [13], ML-enhanced [93], CompressedBF [44], LearnedBF [136], ML-index [45], HM-index [199], RSMI [160], SLBRIN [198]
Reinforcement Learning	QD-tree [214], MTO [47], RLR-tree [72], Waffle [38], BMT [112], WISK [181], ACR-tree [83], PLATON [213]



Learned Multi-dimensional Indexes: Summary

- **Benchmarking**

- Less studied compared to its one-dimensional counterpart
- A recent experimental survey provides an initial benchmark.
 - Liu, Qiyu, Maocheng Li, Yuxiang Zeng, Yanyan Shen, and Lei Chen. "How good are multi-dimensional learned indexes? An experimental survey." The VLDB Journal 34, no. 2 (2025): 1-29.

- **Towards Practical Systems Integration**

- Less explored compared to its one-dimensional counterpart

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Future Research Directions

- **The Lack of Total Ordering for the Multi-dimensional Case**
 - Challenging to provide an error bound in case of ML model mispredictions
- **Choice of ML models**
 - Need to reduce the impact of ML model training time, storage overhead, and prediction latency
- **ML Model Re-training**
 - Significant changes in the underlying input data/query distribution should be detected as soon as possible.
 - Full vs. Partial model re-training

Future Research Directions

- **Supporting Dynamic Inserts/Updates**
 - Further research is needed to investigate the advantages and disadvantages of each approach.
- **Supporting Concurrency**
 - Need to consider the issue of concurrency as a first-class citizen while designing learned indexes
- **Index Compression**
 - Need to further explore the potential benefits of index compression using learned indexes

Future Research Directions

- **Security**
 - The issue of security in the context of learned indexes is little explored.
- **Benchmarking for the Multi-dimensional Case**
 - Extensive benchmarking studies is needed
- **Theoretical Analysis for the Multi-dimensional Case**
 - Analysis of the various components of learned multi-dimensional indexes is needed
- **Leveraging Modern Hardware for Learned Indexes**
 - Natively implementing learned indexes over modern hardware platforms

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THANK YOU